COLLATERAL, TWO-DIMENSIONAL MORAL HAZARD, AND COMPETITION

by

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STATEMENT OF DISSERTATION APPROVAL

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ABSTRACT

This dissertation develops a theory on the use of costly collateral in a credit contract. Existing theories advocate the function of collateral as an incentive device to reduce moral hazard. However, they do not distinguish between the effects of collateral on moral hazard regarding "risk-taking" from those on moral hazard regarding "shirking". I study a model with two-dimensional moral hazard under two extreme market structures: perfect competition and monopoly.

In the section "Risk-taking Moral Hazard" (Section 5), I isolate the effects of risk-taking from those of shirking. I also study adverse selection in which consumer has one of two types (good types and bad types). In the model, banks choose not to use collateral to mitigate risk-taking moral hazard. In a perfectly competitive market, there is a unique separating equilibrium with zero collateral for the bad-type borrower and positive collateral for the good-type. I show that the latter is caused by adverse selection. In the pure monopoly case, zero collateral is optimal for both types.

In the section "Shirking Moral Hazard" (Section 6), I consider shirking moral hazard with a continuum of effort levels, but no adverse selection. In this case, the second-best effort level depends on the trade-off between the marginal benefit of effort (in terms of increasing expected return of the agent's project) and the marginal cost (in terms of reducing limited-liability rents). When collateral is not very costly, it is possible that the good-type borrower exerts effort at a level that is higher than the first-best, regardless of the market structure. In addition, a lower cost of collateral and/or more competition can improve Pareto efficiency of social welfare.

In the section "Two-Dimensional Moral Hazard" (Section 7), I extend my results to the case in which both types of moral hazard exist. Regardless of the market structure, I show that collateral is not used to mitigate risk-taking but to encourage a more efficient effort level. But when the optimal collateral to induce effort happens to be higher than the interest rate, collateral also reduces risk-taking as a side effect.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

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SECTION 1

INTRODUCTION

Collateral is a widespread feature in credit contracts. Berger and Udell (1990) note that "collateral is an important part of more than 70% of commercial and industrial loans made in the U.S.". Steijvers and Voordeckers (2009) show an increasing trend in the use of collateral. The increasing importance of collateral has motivated numerous empirical studies, and yet not much progress has been made in the theoretical literature during the last decade. Stiglitz and Weiss (1981) and Boot, Thakor, and Udell (1991) remain the seminal theoretical papers that empirical studies cite as motivation for their hypotheses. However, existing empirical evidence regarding the function of collateral is mixed.[1] The disparity between theoretical and empirical research of collateral motivates this thesis.

I develop a theory that provides new testable implications and helps us understand the mixed evidence in the empirical research. To study the incentive function of costly collateral in a credit contract, I investigate a two-dimensional moral hazard model under two extreme notions of market structures: perfect competition and pure monopoly. My model has five frictions: costly effort (shirking), limited liability (risk-taking), costly collateral, monopoly power, and adverse selection (with two types of borrowers: good types and bad types). In different sections, I consider different combinations of frictions.

The model suggests that banks use collateral primarily to encourage increased effort to boost productivity. In particular, the cost of collateral has a comprehensive impact on effort so that some borrowers (bad types) exert effort below the first-best level in equilibrium and banks require a low collateral level. In this case, collateral does not affect risk-taking. For other borrowers (good types), effort might be higher than the first-best level and banks require a high collateral level. In this case, collateral has a side effect of controlling risk-

[1] For example, Jimenez, Salas, and Saurina (2006) document a positive association between borrower type and collateral requirements. Berger and Udell (1990) and Chakraborty and Hu (2006) document a negative association, and yet Cressy and Toivanen (2001) find no association at all between borrower type and collateral requirements.

taking. Understanding the exact incentive mechanism of collateral helps banks design more efficient contracts and possibly provides policy guidance in credit markets.

This thesis differs from the existing works in three key respects. First, I separate the effects of moral hazard into two types: the "shirking" effect that mainly affects the expected return of the agent's project, and the "risk-taking" effect that increases project risk without altering its expected return. My model suggests that collateral attenuates shirking moral hazard, while inhibiting risk-taking is only a side effect. Second, unlike prior literature in which effort takes only two levels, I allow for a continuum of effort levels. The resulting comparative statics differ significantly from the existing literature.[2] For example, the relation between the cost of collateral and the equilibrium collateral level is not always monotonic (as in Boot, Thakor, and Udell (1991)). Third, I find that in order to induce a given effort level, higher collateral is needed when a bank has monopoly power. In equilibrium, both effort level and collateral level are higher under monopoly than those under perfect competition. Since collateral is socially costly, competition enhances social efficiency.

The rest of the thesis is organized as follows. Section 2 briefly reviews related papers in the literature. Section 3 describes the main model of two-dimensional moral hazard under different market structures. Section 4 provides a benchmark of what happens in the first best.

In Section 5, I separately investigate the risk-taking friction. I show that because collateral is costly, it is not an efficient incentive mechanism to reduce risk-taking in equilibrium. Due to limited liability, the borrower tends to choose a risky project so that he can enjoy a high residual value when the project is successful. If the project fails, banks bear the costs. In order to mitigate risk-taking, the collateral requirement must exceed the gross interest rate.[3] Otherwise, the borrower still prefers to take more risks because of the fixed expected return from the project.

I also study adverse selection in which borrower has one of two types: good types and bad types. My model implies that banks use collateral to distinguish good types from bad types. In the competitive equilibrium, the good-type borrower accepts positive collateral while the

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[2] I am able to replicate results in Boot, Thakor, and Udell (1991) by allowing only two levels of effort choices and using their assumptions in my model.

[3] In my model, loan size is normalized to \$1, and therefore collateral can be interpreted as collateral amount over loan size. I refer to it as "collateral" hereafter.

bad-type continues to choose a zero-collateral contract. That is, the adverse-selection effect dominates the risk-taking effect. Whereas in the pure monopoly case, zero collateral is optimal for both types of borrowers because collateral is not an efficient signal in this case. As a consequence, monopoly power reduces costly signal of collateral and thus increases social efficiency.

Another implication of the risk-taking model is that risk-taking can be favored in equilibrium as long as it does not destroy value.[4] In other words, using collateral to control risk-taking is not necessarily efficient to the society as a whole, not to mention that collateral itself can be costly. However, in this section, I assume that borrower's risk-taking actions do not affect the project's expected return. Whereas in practice, borrowers might choose a risky project with a low expected return.[5] I address this issue in the next two sections.

In Section 6, I develop a model of shirking that complements existing theories of moral hazard and costly collateral by allowing borrowers to choose among a continuum of effort levels. Regardless of the market structure, my model confirms that banks use collateral to induce higher effort. However, the equilibrium collateral level depends on a bank's trade-off between the marginal benefits and the marginal costs of collateral. On the one hand, collateral serves as a punishment mechanism to induce higher effort and to reduce the borrower's limited-liability rents. On the other hand, collateral introduces inefficiency because it is costly. In addition, the social cost of collateral plays two roles in the benefit-cost trade-off. The first role is to make collateral less effective in reducing the limited-liability rents which calls for decreasing effort below the first-best. The second role is to induce higher effort of the borrower to reduce default risk so that the expected loss of collateral value is minimized. In equilibrium, the dominant effect is determined by different sets of parameters. Therefore, the relation between the equilibrium collateral level and borrower's actions (or types) is not as simple as shown in the prior literature. Rather, it depends on the interactions of different frictions. I show that, when the social cost of collateral is moderate, all else equal, good-type borrowers are more likely to pledge a higher (than the gross interest rate) collateral level and exert effort at an above first-best level. The intuition is that when

[4]This result is partially driven by the assumption of risk-neutrality. But even if I relax this assumption, we can normalize everything to a risk-neutral world and it can be shown that risk-aversion is a second-order effect. Therefore, assuming risk neutrality sets up a theoretical benchmark.

[5]In a world where banks are risk-averse, this is equivalent to the case when the borrower takes a negative NPV project.

collateral is not very costly, it is more effective in inducing higher effort from good-type borrowers, and therefore banks use higher collateral for the good types in equilibrium.

In addition, I find that with moral hazard but no adverse selection, competition enhances social efficiency. The intuition is that when a bank has monopoly power, high collateral is required to induce high effort which increases the expected return of the borrower's project. Nevertheless, in the case of perfect competition, banks must lower the collateral level in order to compete with other banks. As a result, the social loss of collateral is lower under perfect competition than under monopoly so that competition increases social efficiency.

In Section 7, I combine the two types of moral hazard, shirking and risk-taking, into a single model to investigate how collateral interacts simultaneously with both frictions. In the model, borrower's choice of risk-taking affects project risk only, whereas his effort level changes the project's expected return. In order to investigate the issue in practice that the borrower's excessive risk-taking might destroy project value, I assume a complementary relation between effort and risk-control action in determining default risk. I show that most results in the previous two sections still hold. In particular, only in some cases does collateral exceed the gross interest rate and have the side effect of inhibiting risk-taking.[6]

In Section 8, I discuss a few empirical implications with testable hypotheses and propose future work in theory.

The model of costly collateral with two-dimensional moral hazard might also be applied to several other important contracts. For example, in a contract of executive compensation to determine the optimal use of equity and inside debt, the managerial ownership is similar to the gross interest rate in a credit contract, whereas his inside debt position is similar to the collateral requirement. When the manager values the inside debt less than the firm does, inside debt is costly and my model is applicable. It can provide insights in the debate of how firms decide their optimal use of inside debt when contracting is subject to moral hazard. Another application is a loan contract between banks and the central bank. The reserve requirement is equivalent to the collateral requirement, and my model potentially explains why changing the reserve requirement might not be effective in controlling banks' excessive risk-taking.

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[6] Jensen and Meckling (1976) also show that with rational expectation, risk-taking might not imply welfare loss if it does not affect expected return from the investment. They argue that agency costs of debt occur when risk-taking does affect expected return. But they focus on the theory of agency costs and do not study the function of collateral in a debt contract.

Other related issues in a credit contract include alternative mechanisms for a bank to prevent borrowers' value-destroying risk-taking. The most direct method is to enhance monitoring. For example, banks might impose some restrictions on the choice of investment projects indirectly through debt covenants (Rajan and Winton (2010)) or through lending relationships (e.g., Berger and Udell (1995), Lehmann and Neuberger (2001), Chakraborty and Hu (2006)).[7] An indirect method could be to use other corporate contracts that involves the decision maker of the borrowers, such as the CEO. Edmans and Liu (2010) argue that a CEO's pensions and other deferred contingent benefits can be viewed as an inside debt that aligns his interest with the lender's and reduces borrowers' excessive risk-taking behaviors. But this is a topic on the interactions among varied contracts in the corporation (in this case, between compensation contracts and debt contracts), and it is beyond the scope of this thesis.

[7]Rajan and Winton (2010) also argue that collateral can be used as a monitoring mechanism. But that is beyond the scope of this thesis.

SECTION 2

RELATED LITERATURE

This thesis is closely related to two strands of the literature: that of moral hazard in a credit contract (e.g., Jensen and Meckling (1976), Stiglitz and Weiss (1981), Chan and Thakor (1987), Boot, Thakor, and Udell (1991)), and that of how competition interacts with other frictions and affects social efficiency in equilibrium (e.g., Besanko and Thakor (1987a), Chan and Thakor (1987), Villas-Boas and Schmidt-Mohr (1999)).[1] The distinct feature of my model is that I consider two-dimensional moral hazard under different market structures and discuss the implications on the use of collateral in a credit contract environment.

The problem of moral hazard has long been recognized in the credit market (Stiglitz and Weiss (1981)) as well as in the labor market (Holmstrom (1979)). In his 1979 paper, Holmstrom describes moral hazard as a problem that arises when agents' privately taken actions affect the probability distribution of the outcome. Although actions might affect the outcome by changing risk as well as expected return, Holmstrom (1979) focuses on the effect on expected return as the main concern in the labor market is that agents might shirk due to costly actions. He shows that the principal can induce desirable actions by sharing profits with the agent in a risk-sharing contract.

In the credit market, collateral is used as another form of risk-sharing device to mitigate the shirking problem. Instead of profit-sharing, collateral enforces loss-sharing in the event of default. As a result, collateral induces higher effort from the borrower and reduces default risk. However, due to limited liability, the borrower receives all the profits if the project is successful and yet banks bear all the costs in default. Therefore, the borrower has risk-taking incentives even in the present of collateral, unless the collateral requirement is higher than the gross interest rate. In summary, there are at least two types of moral hazard in the credit market: shirking and risk-taking.

[1] Of course, the moral hazard problem is also widely studied in other markets such as the labor market (e.g., Holmstrom (1979), Holmstrom and Milgrom (1987), Holmstrom and Milgrom (1991), Edmans and Liu (2010)). In the sense that I consider two types of actions that are interacted in a model, this thesis is also related to the literature of moral hazard with multiple tasks as in Holmstrom and Milgrom (1991).

However, papers in credit markets are vague in the types of moral hazard. As a result, the function of collateral on moral hazard is not examined precisely. For example, in a two-period model, Stiglitz and Weiss (1981) propose that collateral serves as a punishment mechanism to prevent borrowers from taking too much risk.[2] Later on, in order to incorporate the adverse-selection problem, Chan and Thakor (1987) use a one-period model instead and confirm the incentive function of collateral. But they are both silent about the types of moral hazard.

Stiglitz and Weiss (1981) are among the first to describe risk-taking moral hazard in the credit market: "higher interest rates induce firms to undertake projects with lower probabilities of success but higher payoffs when successful."[3] They show that collateral serves not only as a screening device for the good-type borrowers to distinguish themselves from the bad-types, but also as an incentive device to reduce moral hazard. However, they study adverse selection and moral hazard separately, with the same variable of borrower type. Particularly, they model moral hazard by assuming that the project of a bad-type borrower is riskier than that of a good-type one in the sense of Mean-Preserving-Spreads (MPS).

Nevertheless, it is action but not type that is directly related to moral hazard. Traditional models of contract theory usually assume exogenous variable of type to model adverse selection and use endogenous variable of action to model moral hazard. Chan and Thakor (1987) and Boot, Thakor, and Udell (1991) define a separate variable of effort and combine adverse selection and shirking moral hazard in a single-period model. Chan and Thakor (1987) assume costless collateral and discuss the effect of competition, whereas Boot, Thakor, and Udell (1991) assume costly collateral but only investigate the case of perfect competition. Both papers imply that the first-best effort level is achieved when there is shirking moral hazard. They also find that all borrowers are required to provide collaterals in most of the cases. This result is widely cited, especially in the empirical literature of collateral (see Steijvers and Voordeckers (2009)), as the theoretical backup of the incentive

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[2] While Stiglitz and Weiss (1981) does not consider imperfect competition, Petersen and Rajan (1995) study a similar two-period model of moral hazard assuming a bank has monopoly power. They use the interest rate that is charged by a bank as a proxy for the bank's monopoly power. But they focus on the negative effect of competition on the value of lending relationship.

[3] They also call it the "adverse incentive effect". A similar concept is also raised by Jensen and Meckling (1976) as the "asset substitution effect" or "agency cost of debt." In contrast, the other type of moral hazard, the shirking problem, is similar to "agency cost of equity." Edmans and Liu (2010) also mention the two types of moral hazard, but under the context of CEO compensation contracts.

function of collateral to reduce risk. However, their results are based on two assumptions. First, effort takes only two levels. Second, both effort and type improve a project's return distribution in the sense of First-order Stochastic Dominance (FSD) which captures only the effect of shirking.

This thesis relaxes these two assumptions and re-examines the incentive function of costly collateral. By allowing for a continuum of effort levels, I show that the first-best effort level might not be achieved in equilibrium in most cases. In addition, collateral is primarily used to induce higher effort, but it might not affect risk-taking, and even if it does in some cases, it is only a side effect.

Boot, Thakor, and Udell (1991) also note that with shirking but no adverse selection, the bad-type borrower pledges higher collateral than the good-type in a competitive equilibrium. This prediction has been cited as the theory of the relation between collateral and borrower type regarding the incentive effect of collateral to reduce risk. However, the empirical evidence is mixed. Steijvers and Voordeckers (2009)'s review paper has summarized several papers of mixed results but attribute them to the different effects of moral hazard versus adverse selection. They try to fill in the gap between theory and empirical studies from the empirical side, whereas this thesis does it from the theoretical side. In fact, Boot, Thakor, and Udell (1991) have stated in their paper that the negative relation between collateral and borrower type relies on the assumption that action (effort) and quality (type) are substitutes in determining default risk.[4] They also discuss the difference between "default risk" and "project variance" and caution that their results can be sensitive to the choice of risk measures. However, some follow-up papers choose to ignore these statements and do not construct their hypotheses precisely.

In contrast, my model shows that when effort and quality are complementary in determining default risk and collateral is not very costly, the good-type borrower pledges higher collateral than the bad-type in equilibrium in most cases because their net marginal cost of collateral is relatively lower. The implication is that the relation between collateral and borrower type depends on conditions including the cost of collateral and how effort and quality affect each other. This result might also provide guidance in the empirical research of collateral (see Section 8).

[4]That is, the bad-type borrower's effort is more effective than the good-type's in reducing default risk. In their Footnote 2, they mention that the case of a complementary relation is interesting as well, but they do not explain why they focus on a substitutable relation instead. In Section 6, I show that this is not a key assumption in my model.

In addition, Section 5 of this thesis also investigates the adverse-selection effect in the credit market. I use the standard screening model setting as in Rothschild and Stiglitz (1976).[5] The literature of adverse selection is widely used in different contract settings such as the insurance market, the product market (e.g., Rochet and Stole (1997)), the labor market(e.g., Spence (1973), Lewis and Sappington (1989), Jullien (2000), Guerrieri, Shimer, and Wright (2010)), and the credit market (e.g., Stiglitz and Weiss (1981), Wette (1983), Besanko and Thakor (1987b), Bester (1985)). But in this thesis, I only study it in Section 5 as an extension. More interesting extensions are discussed in Section 8.

The second strand of literature involves models that combine multiple market frictions, such as moral hazard and competition. "Perfect competition" is a key assumption that is widely adopted in models in finance. The prediction of a perfectly competitive credit market is that, if a bank increases its interest rate by a small amount, borrowers switch to another bank immediately. However, we do not see this happen in practice. In fact, anecdotal evidence suggests that some local banks do have monopoly power and dominate the local loan market. More and more papers study imperfect competition in some financial markets. Some papers (e.g., Besanko and Thakor (1987a), Chan and Thakor (1987)) address the effects of competition by providing equilibrium results under two extreme market structures: perfect competition and pure monopoly.[6] Other papers (e.g., Villas-Boas and Schmidt-Mohr (1999)) use the Hoteling setting to discuss the case of imperfect competition.[7]

In this thesis, I investigate equilibria under the two extreme notions of competition. I study the combined effects of risk-taking, adverse-selection and monopoly power and show that zero collateral is optimal for both types of borrowers when a bank has monopoly power. This result is consistent with Besanko and Thakor (1987a) in which they study adverse selection under pure monopoly. The implication is that competition might decrease social welfare when there is no credit-rationing.[8] Villas-Boas and Schmidt-Mohr (1999)

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[5]Rothschild and Stiglitz (1976) focus on a perfectly competitive insurance market. For the pure monopoly market, I follow Baron and Myerson (1982).

[6]Mussa and Rosen (1978), Maskin and Riley (1984) and Rayo (2005) are examples in the product market. Example in the labor market includes Boal and Ransom (1997), Bhaskar and Manning (2002), Manning (2003a), and Manning (2003).

[7]The literature of imperfect competition can be traced back to Hotelling (1929) and Robinson (1933). Rochet and Stole (2002) and Diaz-Diaz and Rayo (2009) are examples of its application in the product market.

[8]While credit rationing is important in a model with adverse selection, this thesis focuses on moral

also find a similar result, but they use the Hoteling setting instead to study the effect of imperfect competition.

Chan and Thakor (1987) combine adverse selection, shirking moral hazard and different market structures. They show that competition reduces distortions in equilibrium. However, they assume costless collateral and focus on the distortion of credit rationing. They also mention that their results rely on the assumption of costless collateral and results might change if assuming costly collateral. In addition, they do not consider risk-taking moral hazard. In contrast, my model assumes costly collateral and consider the interaction between two-dimensional moral hazard and competition. Besanko and Thakor (1987a) and Villas-Boas and Schmidt-Mohr (1999) also assume costly collateral but focus on the adverse-selection problem. They show that monopoly power eliminates signalling and thus lower the equilibrium collateral level and improves social efficiency. This thesis complements the existing literature by studying the case when there is moral hazard but no adverse selection. My model predicts that more competition improves social efficiency because competition lowers collateral requirement and thus reduces the expected costs of collateral. Table 2.1 compares models in different papers with different sets of frictions.

Table 2.1: Literature and Comparison

Paper	**Cost of Collateral**	**Market Structure**	**Effort Choices**	**Adverse Selection**
Besanko and Thakor (1987a)	both ($0 \le \beta \le 1$)	competitive monopoly	NA NA	Yes Yes
Chan and Thakor (1987)	costless ($\beta = 1$)	competitive monopoly	2 2	Yes Yes
Boot, Thakor, and Udell (1991)	costly ($0 \le \beta < 1$)	competitive	2	Yes
Villas-Boas and Schmidt-Mohr (1999)	costly ($\beta = 0$)	imperfect competition	NA	Yes
This Thesis Section 5	costly ($0 \le \beta < 1$)	competitive monopoly	continuum continuum	 Yes

hazard and credit rationing is only a second-order effect.

SECTION 3

MODEL

I study a principal-agent relationship between risk-neutral banks (principals) and a risk-neutral borrower (agent) in the credit market, where the agent takes two private actions e (effort) and a (risk-taking). These actions affect the outcome distribution of a project. There is one period with three stages, as shown in Figure 3.1.

In the first stage, each bank offers a \$1 credit contract that requires a collateral level and a gross interest rate (C, r) (r is one plus interest rate), and the borrower either accepts or refuses the contract. If the contract is accepted by the borrower, they move on to the second stage. Otherwise, no contract is signed.

In the second stage, provided the borrower accepts the contract, he chooses a project and take two private actions $e \in R^+$ and $a \in R^+$. The outcome distribution of the project depends on the endogenously determined actions e and a as well as the borrower's quality type that is exogenously given as $\theta \in R^+$. For tractability, I assume the project only has two outcomes: success and failure.[1] Upon success, the project yields a return of $\dfrac{eL\theta}{p(e,a,\theta)}$,

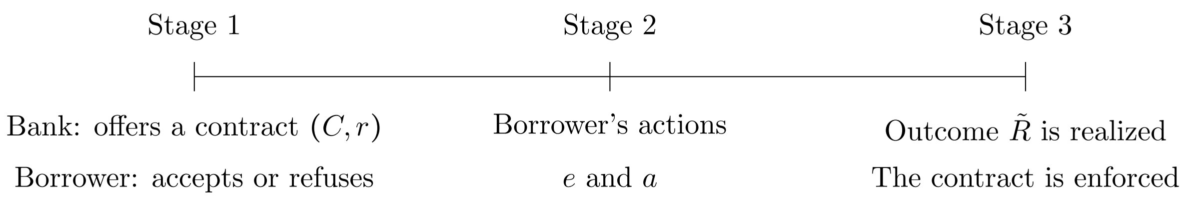

Figure 3.1: Timing of Model

[1]This two-outcome technology has been widely used in the literature of credit markets to simplify the calculation and retrieve analytical solutions for complex screening models, such as models with multiple market frictions (Stiglitz and Weiss (1981), Besanko and Thakor (1987a)). Especially, like Besanko and Thakor (1987a) and a series of follow-up papers, I set the project's return in the event of default to be zero so that the model is tractable without changing the basic intuitions. Of course, this assumption is not perfect because a project usually pays off something even if it fails in the real world. But what matters in my analysis is the difference between the two outcomes, and any real life outcomes can be normalized to the setting here.

where L is a positive constant and $p(e, a, \theta)$ is the probability of success. If it fails, it pays nothing.

To summarize, the project's return is:

$$\tilde{R} = \begin{cases} \dfrac{eL\theta}{p(e,a,\theta)} & \text{with probability } p(e,a,\theta) \\ 0 & \text{with probability } 1 - p(e,a,\theta) \end{cases}$$

Finally, in the third stage, the outcome of the project is realized and the contract is enforced by court. If the project is successful, the borrower pays interest r to the bank and keeps the project's return $eL\theta/p(e,a,\theta)$ (and does not lose the collateral C), while the bank receives a fixed payment of r. If the project fails, the borrower pays no interest but gets nothing from the project and loses the collateral C, whereas the bank receives only a discounted value of collateral βC $(0 \le \beta < 1)$. Regardless of the project's outcome, the borrower's effort e costs him $D(e)$, which is an increasing and convex function in e. Similarly, the borrower's action a costs him $D(a)$, which is also an increasing and convex function in a.

My first set of assumptions includes basic assumptions that I will use throughout.

Assumption 1 (A1)

a. *For any given θ, $p(e,a,\theta)$ is smooth and strictly increasing in both arguments.*

b. *Effort e lies in a compact interval $[0, \overline{e}]$, and action a lies in the interval $[\underline{a}, \overline{a}]$ where $\underline{a} > 0$. Moreover, any available choices of e and a satisfy $0 < p(e,a,\theta) \le 1$.*

c. *The borrower has unlimited access to collateral.*[2] *Moreover, $L\theta$ is large enough to insure a non-negative utility for the borrower.*

d. *Collateral is costly: $0 \le \beta < 1$.*

The distribution of the project's return $\tilde{R}$ and $(A1a)$ together imply that, given a borrower of type θ, for a given action a, a higher effort level e improves the return distribution in a first-order stochastic-dominance (FSD) sense, whereas, for a given effort e, a higher action a improves the return distribution in a mean-preserving-spread (MPS) sense. Particularly, for a given action a, a higher e increases not only the probability of success but

[2] In their Footnote 6, Chan and Thakor (1987) offer a story of (premature) costly liquidation of collateral to explain the reason why a loan is needed even with unlimited collateral available. Also see Besanko and Thakor (1987a) for a discussion of an adverse selection model with limited collateral.

also the expected return from the project, whereas for a given effort e, a higher a increases the probability of success without affecting the expected return. This setting allows a separation of risk-taking from shirking. All else equal, a higher action a corresponds to a less risky project, no matter whether it is measured by default probability or project variance.[3] Further, under this setting, the borrower can affect the expected return of the project, regardless the value of a, by changing e.[4]

$(A1c)$ prevents a credit-rationing problem, thus allowing me to focus on the function of collateral regarding its effects on both dimensions of moral hazard. Moreover, Section 5, where I consider an adverse-selection problem, this assumption allows me to focus on the function of collateral as a sorting mechanism. In order that $(A1c)$ holds, $L\theta$ must be sufficiently large.

$(A1d)$ is a crucial assumption. β measures how banks value collateral relative to how the borrower does. $\beta = 1$ means both parties agree on the value of collateral. When $0 \leq \beta < 1$, they value the collateral differently: the borrower values the collateral more than banks. Consequently, when the project fails, the borrower loses C whereas a bank gets only βC, generating a dead-weight-loss that is equal to $(1 - \beta)C$.

I impose Assumption $(A1d)$ for two reasons. First, when $\beta = 1$, a simple contract with $C = r = 1$ can trivially eliminate the moral hazard problem.[5] Second, in reality, collateral is costly in two respects. On the one hand, when a project fails, a bank needs to liquidate collateral and convert it into cash, leading to a liquidation cost. On the other hand, collateral might sometimes be firm-specific so it is worth more to the borrower than to any other participations in the asset market (including banks).

In each section below, I will impose further assumptions to simplify my analysis.

Table 3.1 summarizes the variables and payoffs of the model.

A bank's expected profit from a credit contract (C, r) for a type θ borrower is given by:

$$\Pi(C, r; e, a, \theta) = p(e, a, \theta) \cdot r + (1 - p(e, a, \theta)) \cdot \beta C - 1. \tag{1}$$

---

[3] Boot, Thakor, and Udell (1991) discusses a potential conflict in ranking risk using the two measures of risk: default risk and project variance. This conflict does not exist here with my assumptions on action a.

[4] In contrast, Stiglitz and Weiss (1981) assume that type affects the distribution of the project's return in a MPS sense, while Chan and Thakor (1987) and Boot, Thakor, and Udell (1991) consider type and effort both in a FSD sense.

[5] Chan and Thakor (1987) study shirking with $\beta = 1$ but when there is adverse selection.

Table 3.1: Variables and Payoffs of the Model

Scenario	Bank	Borrower i	Total Surplus
Success:	$r-1$	$\dfrac{eL\theta}{p(e,a,\theta)} - r - D(e) - D(a)$	$\dfrac{eL\theta}{p(e,a,\theta)} - 1 - D(e) - D(a)$
Failure:	$\beta C - 1$	$-C - D(e) - D(a)$	$-(1-\beta)C - 1 - D(e) - D(a)$

The expected utility of a type θ borrower is given by:

$$V(C,r;e,a,\theta) = eL\theta - p(e,a,\theta)\cdot r - (1-p(e,a,\theta))\cdot C - D(e) - D(a). \tag{2}$$

The expected total surplus of the bank and a type θ borrower, given contract (C,r), is given by:

$$S(C;e,a,\theta) = \Pi(C,r;e,a,\theta)+V(C,r;e,a,\theta) = eL\theta-(1-p(e,a,\theta))\cdot(1-\beta)C-1-D(e)-D(a). \tag{3}$$

In what follows, I define the equilibria of this model under two extreme notions of competition among banks: perfect competition and pure monopoly.[6]

Perfect competition among banks produces credit contracts that maximize the borrower's expected utility subject to two constraints: for a given type θ, each bank makes a non-negative profit and the borrower has a non-negative utility.[7] That is:

$$\Pi(C,r;e^*,a^*,\theta) \geq 0, \tag{IR_Π}$$

and

$$V(C,r;e^*,a^*,\theta) \geq 0, \tag{IR_V}$$

where e^* and a^* are the equilibrium effort and action given the contract C and r, for a given type θ.

In equilibrium, each bank earns a zero profit. The reason is that, in order to earn a positive profit, a bank has to charge a higher interest rate or higher collateral than its competitors. However, when there is perfect competition, this will drive all its customers away.

[6]The definitions here are for models with moral hazard in general. But in Section 5, because I also consider adverse selection, definition of equilibria is restated following Rothschild and Stiglitz (1976).

[7]This notion of competition is similar to the T1 competition as defined in Chan and Thakor (1987).

In the pure monopoly case, in contrast, the principal's program generates credit contracts that maximize a bank's expected profit subject to the borrower's participating constraints (IR_V).[8] In this case, it is the (IR_V) constraint that always binds in equilibrium.

In Section 4, I derive the first-best allocation, which provides a benchmark of social efficiency. Section 5 and 6 study each type of moral hazard separately. In Section 7, I consider both type of moral hazard jointly.

[8]This notion of competition is slightly different from that in Chan and Thakor (1987). Whereas they consider competition among banks for a limited quantity of deposits, I choose to focus on the monopoly power of a bank in the credit market. In particular, they assume that the deposit interest rate is endogenously determined so that the bank's equilibrium expected profit is always zero. However, I assume that the cost of deposit funding is fixed at \$1 (with a zero interest rate) so that a bank has monopoly power in the credit market and can make a positive profit.

SECTION 4

FIRST BEST

Suppose the agent can self finance, so that there is no conflict of interest.[1] I now consider the allocation of (C, e, a) to maximize the borrower's value (and therefore the total surplus). I call this allocation the first-best allocation.

Lemma 1 *The first-best allocation involves zero collateral ($C^{FB} = 0$), an effort e^{FB} such that:*

$$L\theta = D'(e^{FB}), \tag{4^{FB}}$$

and an action $a^{FB} = \underline{a}$ if $D(a) > 0$ and a^{FB} takes any value otherwise.

It is not surprising that zero collateral is optimal under first best, given the assumption of costly collateral. With zero collateral, the first-best effort e^{FB} reflects the trade-off between a higher return and the disutility of higher effort. By assumption, risk-taking does not affect the expected return of the agent's project, so the first-best risk-taking depends solely by its costs.

In the next section, in order to obtain a general result, I assume that $D'(a) > 0$ (namely, reducing risk-taking is costly) for all a. In this case, the first-best risk-taking is $\underline{a}$. I also show that the second-best results with risk-taking and adverse selection hold even with $D'(a) = 0$. In Section 6, in contrast, I assume $D'(a) = 0$ (so that reducing risk-taking is costless) for all a. Without loss of generality, I normalize $D(a)$ to be equal to zero to simplify my analysis (see Footnote 2 in Section 7). In this case, the first-best risk-taking a can be any value.

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[1]Equivalently, we can assume, for this section, effort e and action a are contractible.

SECTION 5

RISK-TAKING MORAL HAZARD

In this section, I combine risk-taking moral hazard (without shirking) and adverse selection. My second set of assumptions is just for this section. These assumptions isolate the risk-taking effect and introduce adverse selection.

Assumption 2 (A2)

a. There is no shirking problem: e always equals to 1 and $D(1) = 0$.

b. There are two types of borrowers: a good-type (θ_G) and a bad-type (θ_B), with $\theta_G > \theta_B$.

c. The cost of controlling risk-taking is $D(a) = 1/2\gamma a^2$ where $\gamma > 0$.[1]

d. The probability of success of the agent's project is $p = a_i\theta_i$ ($i \in \{G, B\}$).

I assume $e = 1$ so that the expected return of the project is fixed at $L\theta_i$ for any given θ_i. But later in Section 7, I relax this assumption.

To model adverse selection, I use the standard screening setting (as described in Rothschild and Stiglitz (1976)). Each bank offers a menu of \$1 credit contracts (C, r). Each borrower knows his quality type θ_i and chooses a contract from the menu to finance a project. But θ_i is unknown to banks before contracting. In this section, I consider a simple example of two types of borrowers as stated in Assumption ($A2b$): the good-type (with θ_G) and the bad-type (with θ_B).[2]

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[1] I generally assume that $\gamma > 0$ unless otherwise specified and that γ is not too big so that the borrowers always get non-negative utilities. But of course, $\gamma = 0$ is an interesting case in which risk-taking (that can be generated with a low a_i) is absolutely good for the social welfare in a risk-neutral world.

[2] Prior literature has different conclusions regarding whether the equilibrium results can be extended to the case with a continuum of types. Rothschild and Stiglitz (1976) discuss in their Footnote 7 about the non-existence of equilibrium due to the benefit of pooling. However, Chan and Thakor (1987) argue that "the results with a continuum of types will be mostly similar to those obtained here" (either separating or pooling). It is interesting to see whether this difference in finding can be attributed to the introduction of moral hazard effects. But I leave this issue to future research.

With the Assumptions set $(A2)$, the project's return becomes:

$$\tilde{R} = \begin{cases} \dfrac{L}{a_i} & \text{with probability } a_i\theta_i \\ 0 & \text{with probability } 1 - a_i\theta_i \end{cases}$$

Assumption $(A2c)$ and $(A2d)$ are to simplify the analysis so that I can also consider adverse selection. With this setting, the variance of the project's return is given by $Var(\tilde{R}) = L^2\theta_i \cdot (\frac{1}{a_i} - \theta_i)$. In this case, a good-type borrower can take an action to control risk-taking more effectively than a bad-type one.[3].

The following two subsections study the competitive screening equilibrium and the monopolistic screening equilibrium, respectively.

5.1 Competitive Screening Equilibrium

This subsection studies the screening equilibrium in a perfectly competitive credit market, using the Cournot-Nash type equilibrium concept in Rothschild and Stiglitz (1976). Robustness is also established in the demonstrations.

5.1.1 No Frictions

I first consider the Cournot-Nash equilibrium when there are no hidden actions and all borrowers are of the same quality type θ_0. To find the optimal contract in the perfectly competitive credit market, one solves the following program:

$$\max_{C_0, r_0} V(C_0, r_0; a_0^*, \theta_0) = L\theta_0 - a_0^*\theta_0 \cdot r_0 - (1 - a_0^*\theta_0) \cdot C_0 - 1/2\gamma a_0^{*2},$$

$$\text{subject to } \Pi(C_0, r_0; a_0^*, \theta_0) = a_0^*\theta_0 \cdot r_0 + (1 - a_0^*\theta_0) \cdot \beta C_0 - 1 \geq 0.$$

That is, to maximize the borrower's expected utility given that banks make non-negative profits. In Section 5.1.2, I introduce an additional incentive constraint (ICM) to discuss risk-taking moral hazard, and I add another two incentive constraints (ICG) and (ICB) to account for adverse selection in Section 5.1.3.

As I have shown in Section 4, the first-best contract involves zero collateral and highest risk-taking ($a^{FB} = \underline{a}$) if to control risk-taking is costly ($\gamma > 0$). Under perfect competition,

---

[3]This is similar to the "IMRQ" assumption in Chan and Thakor (1987). But Boot, Thakor, and Udell (1991) assume the opposite. I show that my results do not depend on this assumption in Appendix A. I would like to thank Professor Shmuel Baruch for suggesting this interesting exercise.

the optimal interest rate with a first-best contract is then determined by the bank's break-even condition (IR_{Π}): $r_0^* = \dfrac{1}{\underline{a}\theta_0}$, and the borrower's optimal value is thus given by $V_0^* = L\theta_0 - \dfrac{1}{2}\gamma\underline{a}^2 - 1$.

For the purpose of the following analysis, I also illustrate this result graphically. A credit contract (C_0, r_0) is represented in Figure 5.1 with the horizontal axis C and the vertical axis r. F is the competitive equilibrium contract that maximizes the borrower (θ_0)'s utility and makes banks break even.

For any given action a_0, the borrower's indifference curves are represented by the light (green) lines with a slope of $-\dfrac{1-a_0\theta_0}{a_0\theta_0}$ (level sets of the function of equation (2) in Section 3). The utility level increases as the lines move towards the south-west corner, because a lower interest rate and a lower collateral level are good for the borrower. A bank's break-even line is in bold (black) with a slope of $-\dfrac{\beta(1-a_0\theta_0)}{a_0\theta_0}$. Any contract beyond this line makes a non-negative profit for the bank. Because $0 \leq \beta < 1$, a break-even line is always flatter than any indifference curves. As a result, the equilibrium contract F is located at the intersection of the bank's break-even line and the borrower's first-best indifference curve (the lowest light (green) line), and it happens to be on the r axis ($C_0 = 0$).

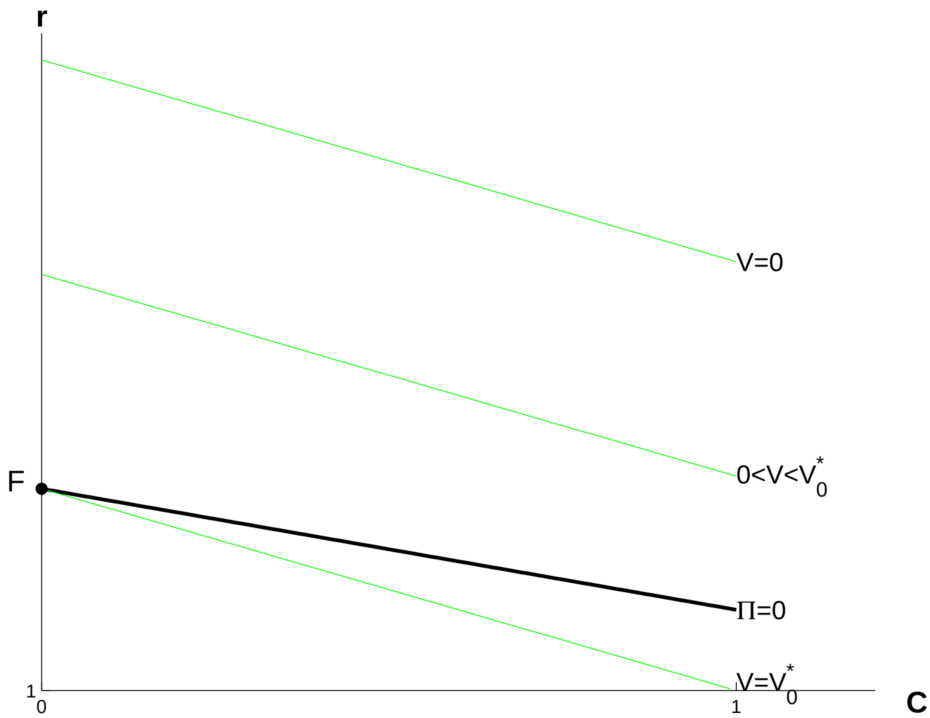

Figure 5.1: Perfect Competition with No Frictions

Equation (2) implies that any borrower's first-best indifference curve would always pass through the point $(C = 1, r = 1)$. At this point, the borrower pays a fixed cost of \$1 and gets the actuarial fair utility which happens to be the same as the first-best utility. Because $r \geq 1$, with a negative slope, the borrower's first-best indifference curve is no longer valid beyond this point. Although the risk-control action a is not directly presented in Figure 5.1, a higher action results in a flatter set of indifference curves and break-even lines with a lower intersection on the r axis. Thus, for every value of a, we have a corresponding Figure 5.1 with corresponding slopes of the curves. Which action to choose depends on the cost of the action: if $\gamma = 0$, a^* can be any value; if $\gamma > 0$, $a^* = \underline{a}$. But with a higher equilibrium action, the intersection on the r axis is lower and so is the equilibrium interest rate.

5.1.2 Risk-taking But No Adverse Selection

I consider the case when borrowers are of the same quality type θ_0 but their risk-taking actions are now unobservable. With hidden actions, moral hazard problem arises. On the one hand, the borrower chooses an action of risk-taking in his best interest given a signed credit contract. He usually chooses to take as much risk as possible because of limited liability. On the other hand, banks would take this into account ex-ante and offer an optimal contract to induce a most favorable action.

In general, the borrower's actions affect the project's return in two aspects: expected return and risk. Prior literature of moral hazard in economics, especially in labor market models, emphasizes the effect on expected return. In those models, the principal desires that agents exert high effort to achieve high expected return. The agents, however, tend to exert lower effort than desired because effort is costly. This shirking moral hazard problem usually can be solved by a profit-sharing contract that allows the agents to enjoy part of the returns or to bear part of the losses and thus induces desirable actions. I discuss the shirking problem in details in Section 6. But in this section, I focus on risk-taking. Mathematically, I employ the Mean-Preserving-Spread (MPS) technology to describe the project's return. The incentive constraint of risk-taking is therefore given by:

$$a_0^* \in \arg\max_{a_0} V(C_0, r_0; \theta_0, a_0) = L\theta_0 - a_0\theta_0 r_0 - (1 - \theta_0 a_0) \cdot C_0 - \frac{1}{2}\gamma a_0^2,$$

or after simplification:

$$a_0^* = \min\{\max\{\underline{a}, \frac{\theta_0}{\gamma}(C_0 - r_0)\}, 1\}. \tag{ICM}$$

Proposition 1 *In a perfectly competitive credit market with risk-taking moral hazard, the first-best contract can be achieved:* $C_0' = 0$, $r_0' = \dfrac{1}{\underline{a}\theta_0}$, *and the optimal action for the borrower is* $\underline{a}$.

Equation (ICM) implies that with risk-taking moral hazard, any action $a_0 > \underline{a}$ is never chosen where there is zero collateral. Because collateral is costly, it looks as if action is also costly (even when $\gamma = 0$) and the borrower would want to choose the lowest action possible. In Figure 5.1, because the first-best utility level is not achievable beyond the point $(C = 1, r = 1)$, $C \leq 1 \leq r$. Together with (ICM), we can see that $\underline{a}$ is the optimal action in equilibrium.

5.1.3 Risk-taking and Adverse Selection

So far I consider the case with borrowers of the same type. Now I consider borrowers of more than one type that are unknown to banks. In this case, each bank offers a menu of contracts to induce truth-telling and desirable actions. Each borrower then chooses his optimal action of risk-taking based on the signed credit contract. However, it is not obvious ex-ante whose (good-type or bad-type borrowers) expected utility would be maximized in equilibrium. I follow the equilibrium concept as defined in Rothschild and Stiglitz (1976) in the analysis.

Equilibrium in a competitive credit market with both risk-taking moral hazard and hidden information is a set of credit contracts such that, when borrowers choose contracts to maximize their expected utilities, (i) no contract in the equilibrium set makes negative expected profits for a bank; and (ii) there is no contract outside the equilibrium set that, if offered, makes a non-negative profit for any bank; and (iii) all the banks offering contracts have taken into account the fact that borrowers choose their optimal actions for investment after contracting.[4]

I consider a simple example with only two types of borrowers ($i = \{G, B\}$): the good-type (θ_G) and the bad-type (θ_B). Suppose that the fraction of the bad-type borrowers is λ, and thus the average quality is $\bar{\theta} = \lambda\theta_B + (1-\lambda)\theta_G$. Compared to the case in Section 5.1.2, two additional truth-telling incentive constraints are required in equilibrium:

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[4]Notice that Rothschild and Stiglitz (1976) consider the insurance market model with adverse selection but without moral hazard problem. Because my model also deals with risk-taking moral hazard, an additional incentive constraint is needed in equilibrium that borrowers choose " their optimal actions for investment after contracting."

$$V(C_G, r_G; \theta_G, a_G^*) \geq V(C_B, r_B; \theta_G, \hat{a}_G^*) \qquad (ICG)$$

$$V(C_B, r_B; \theta_B, a_B^*) \geq V(C_G, r_G; \theta_B, \hat{a}_B^*) \qquad (ICB)$$

where $\hat{a}_i^* \in \arg\max_{\hat{a}_i} V(C_{-i}, r_{-i}; \theta_i, \hat{a}_i)$. That is, $\hat{a}_i^* = \min\{\max\{\underline{a}, \frac{\theta_i}{c}(C_{-i} - r_{-i})\}, 1\}$.

In this case, there are only two possible kinds of equilibria: pooling and separating. In the following analysis, I first show that there cannot be a pooling equilibrium (see Lemma 2 and Figure 5.2). I then derive the separating equilibrium when $C < 1$ and λ is large enough in Lemma 3 (Figure 5.3): the bad-type borrower gets the first-best contract (zero collateral) while the good-type borrower is required to provide positive collateral (upwardly distorted). The condition of λ is also provided to insure the existence of the separating equilibrium (Figure 5.4). Proposition 2 then concludes that this separating equilibrium is unique by demonstrating that there is no sustainable equilibrium when $C \geq 1$.

Lemma 2 *There cannot be a pooling equilibrium.*

Figure 5.2 demonstrates the nonexistence of a pooling equilibrium. The good-type borrower's indifference curve is represented by the (red) solid line, with a slope of $-\dfrac{1 - a_G\theta_G}{a_G\theta_G}$,

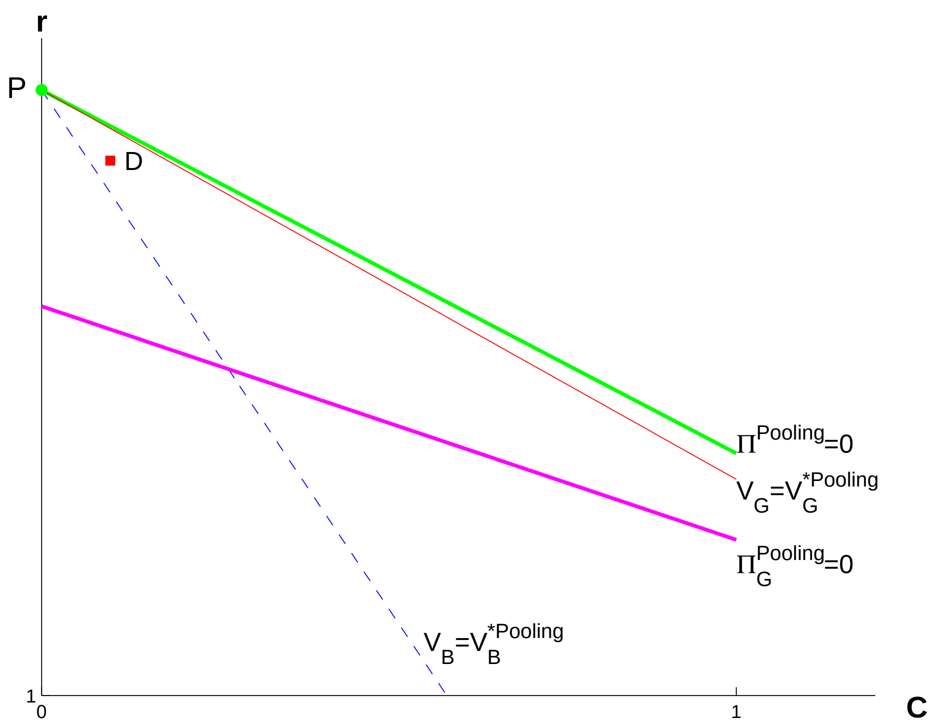

Figure 5.2: No Pooling

while the bad-type one is represented by the (blue) dash line, with a slope of $-\frac{1-a_B\theta_B}{a_B\theta_B}$. Due to the same reason of costly collateral as I showed in Section 5.1.1 and 5.1.2, the optimal pooling contract must involve zero collateral. Thus, Equation (ICM) implies that $\underline{a}$ is the optimal action for both types of borrowers. Suppose the (green) dot P is the pooling equilibrium, it must be on the bank's break-even line $\Pi(C_P, r_P; \bar{\theta}) = 0$. It is the (green) bold line with a slope of $-\frac{\beta(1-\underline{a}\bar{\theta})}{\underline{a}\bar{\theta}}$ (It is also called the "market odds line".) Rothschild and Stiglitz (1976) have described the reason by exhausting alternatives. On the one hand, $\Pi(C_P, r_P; \bar{\theta}) < 0$ apparently violates the definition of equilibrium. On the other hand, if $\Pi(C_P, r_P; \bar{\theta}) > 0$, there is a contract that can slightly increase the expected utilities of both types of borrowers and at the same time make a profit for a bank. It also violates the definition of equilibrium. Thus, the potential pooling equilibrium P is located at the intersection of the bank's break-even line and the indifference curves of both types of borrowers.

However, P is not an equilibrium. For example, the good-type borrower might prefer contract D, the (red) square in Figure 5.2, although the bad-type borrower still prefers P. Because D is near P, banks can still make a profit from the good-type borrower (as long as D is above the (pink) bold line, the bank's break-even line of the good-type contract). That is, in order to maximize its profits, a bank would deviate to a contract like D and attract only the good-type borrower. It disqualifies P from being an equilibrium.

Lemma 3 *When $C < 1$, for any λ that is large enough ($\lambda > \frac{(1-\beta)(1-\underline{a}\theta_G)\theta_G}{(1-\beta)(1-\underline{a}\theta_G)\theta_G + \theta_G - \theta_B}$), there is a unique separating equilibrium where:*
(1) both types of borrowers choose action $\underline{a}$;
(2) the bad-type contract is first best: $C_B = 0$ and $r_B = \frac{1}{\underline{a}\theta_B}$;
(3) the good-type contract is distorted (a positive collateral but a lower interest rate):
$$C_G = \frac{\theta_G - \theta_B}{\theta_G - \theta_B[\beta + (1-\beta)\underline{a}\theta_G]} \text{ and } r_G = \frac{\beta\theta_G + (1-\beta)\frac{1}{\underline{a}} - \theta_B}{\theta_G - \theta_B[\beta + (1-\beta)\underline{a}\theta_G]}.$$

I prove Lemma 3 in two steps.

Step 1: With the help of Figure 5.3, I derive a potential separating equilibrium when $C < 1$.

$C < 1 \leq r$ implies that any borrower's optimal action given any contract within this region is exactly the corner solution $\underline{a}$ (see equation (ICM) in Section 5.1.2).

Following the same notations: the (red) solid lines are for the good-type borrower and

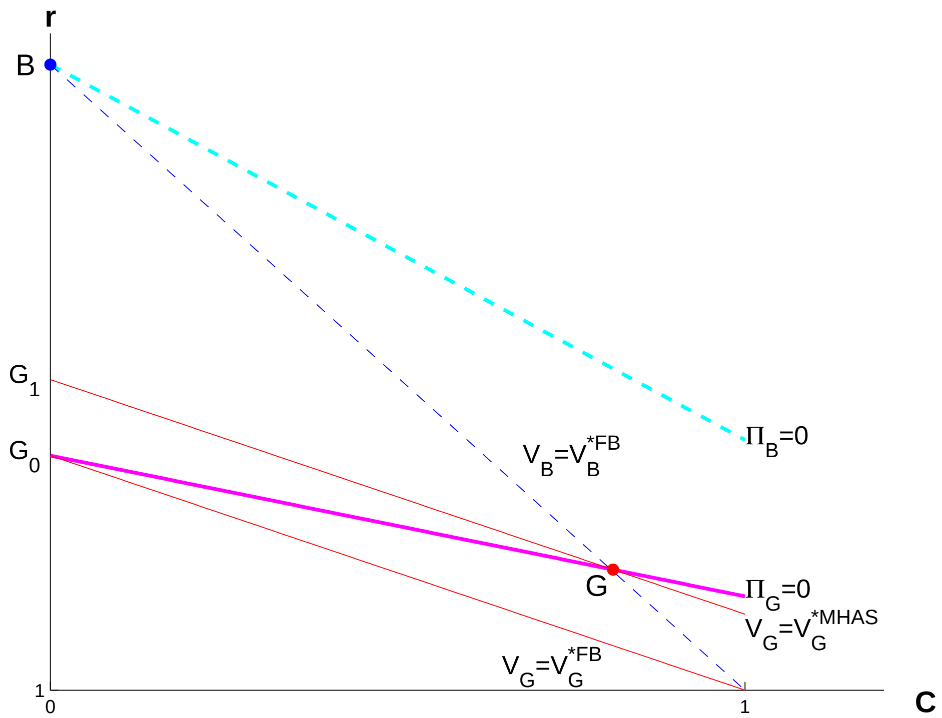

Figure 5.3: Separating

the (blue) dash lines for the bad-type. The bold lines are the zero-profits lines: the (pink) solid one for the good-type contract (with a slope of $-\dfrac{\beta(1-\underline{a}\theta_G)}{\underline{a}\theta_G}$) and the (blue) dash one for the bad-type contract (with a slope of $-\dfrac{\beta(1-\underline{a}\theta_B)}{\underline{a}\theta_B}$). By definition, banks makes zero profits from each contract in equilibrium.[5] In other words, the equilibrium contract for the good-type must lie on the (pink) bold solid line and that for the bad-type must lie on the (blue) bold dash line. Section 5.1.1 has shown that with perfect information, each borrower would most prefer the contract at the intersection of the first-best indifference curve and the zero-profit line. That is, point B for the bad-type and point G_0 for the good-type.

However, this set of contracts B and G_0 cannot be an equilibrium when type is private information. As shown in Figure 5.3, contract B for the bad-type yields a higher interest rate than G_0 for the good-type. As a result, the bad-type borrower would want to mimic the good-type to get better off. In order to induce truth telling, the good-type contract ought to lie on the bad-type borrower's first-best indifference curve. Thus, we have the potential separating equilibrium contract set: B (the blue dot) for the bad-type and G (the

[5]The optimal subsidy argument is discussed in the second step of this proof.

red dot) for the good-type.

Mathematically, this equilibrium results from maximizing the good-type borrower's utilities subject to the incentive constraint (ICB) as well as moral hazard constraint and the bank's zero-profit condition. It can be proved that the optimal good-type contract is obtained with binding (ICB). Intuitively, all else equal, the good-type borrower is willing to provide a collateral that is high enough to prevent the bad-type from mimicing. Therefore, the good-type borrower gets an optimal utility of:

$$V_G^* = l\theta_G - 1 - \frac{(1-\beta)(1-\underline{a}\theta_G)(\theta_G-\theta_B)}{\theta_G-\theta_B[\beta+(1-\beta)\underline{a}\theta_G]} - \frac{1}{2}c\underline{a}^2. \tag{2^G}$$

Step 2: Now that I have identified a potential separating equilibrium, the next step is to demonstrate under what conditions it is sustainable. I explore alternative pooling and separating equilibria that might potentially upset the prior separating equilibrium and show that this is not an issue when λ is large enough. But the details to derive the condition of λ are tedious so I move them to Appendix A.

First of all, a pooling contract like P' in Figure 5.4 is preferred by both types of borrowers and might upset the prior separating equilibrium. In Figure 5.4, the (green) dash-dot lines

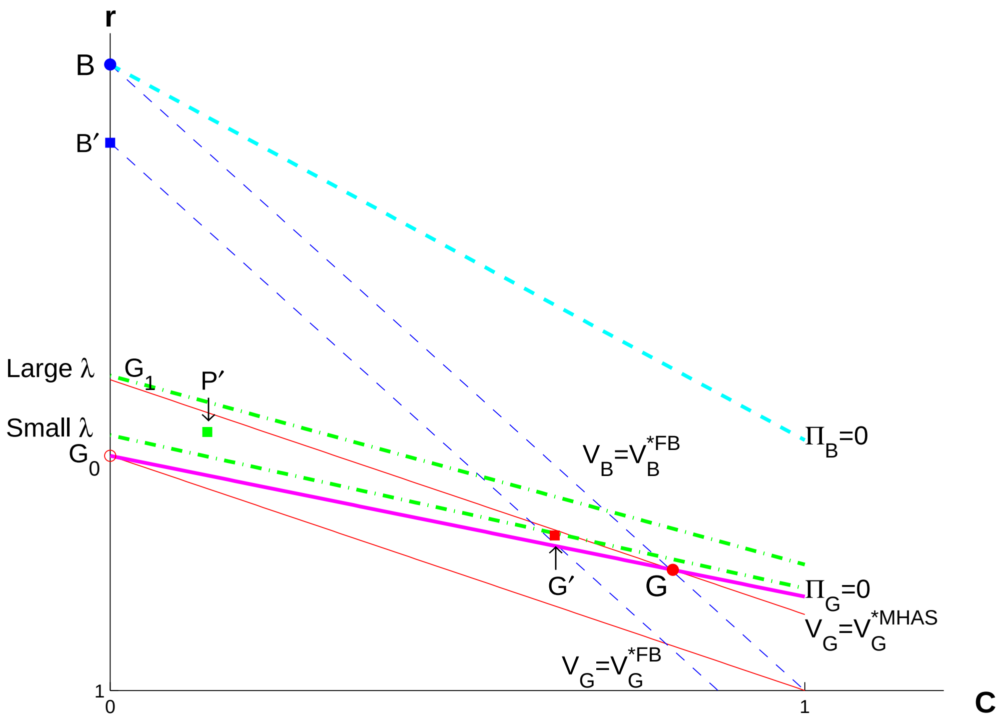

Figure 5.4: Existence

are the market odds lines for a pooling contract P with different λs ($\Pi(C_P, r_P; \bar{\theta}_\lambda) = 0$). With a large enough λ, the market odds line always lies above the indifference curve of the good-type borrower G_1G, and any alternative pooling contract P' that is preferred by both types of borrowers can only located in the negative profit region. In fact, I can show that for any $\lambda > \dfrac{(1-\beta)(1-\underline{a}\theta_G)\theta_G}{(1-\beta)(1-\underline{a}\theta_G)\theta_G + \theta_G - \theta_B}$, no pooling contract can upset the separating equilibrium of B and G. In contrast, if λ falls below this percentage, there might be no equilibrium in the competitive credit market.

Intuitively, when the two types are very different from each other ($\theta_G \gg \theta_B$), as long as there is a small population of the bad-type borrowers, each bank gets a negative profit from a pooling contract. In addition, when collateral is more costly (smaller β), it is more likely to end up with no equilibrium.

Second, when a bank is allowed to offer more than one contract, an alternative contract set B' and G' might upset the prior separating equilibrium. As shown in Figure 5.4, the bad-type borrower would prefer B' to B and the good-type would prefer G' to G. Although a bank makes a negative profit from the bad-type borrower, the loss may be subsidized by the positive profit from the good-type. That is, a bank might choose this alternative contract set if on average they can make a non-negative profit. Similar to Rothschild and Stiglitz (1976)'s optimal subsidy argument, I can also demonstrate that for any $\lambda > \dfrac{(1-\beta)(1-\underline{a}\theta_G)\theta_G}{(1-\beta)(1-\underline{a}\theta_G)\theta_G + \theta_G - \theta_B}$, there is no other separating equilibrium that would make a non-negative profit, because any deviation of the bad-type contract would lead to a loss that is too big to be subsidized by the good-type contract. Interestingly, this condition happens to be the same as the one that rules out alternative pooling equilibrium in **Step 1**.

Proposition 2 *In a competitive credit market with risk-taking moral hazard and adverse selection, when λ is large enough ($\lambda > \dfrac{(1-\beta)(1-\underline{a}\theta_G)\theta_G}{(1-\beta)(1-\underline{a}\theta_G)\theta_G + \theta_G - \theta_B}$), there is a unique equilibrium as shown in Lemma 3.*

In the following, I prove Proposition 2 by showing that no contract with $C \geq 1$ can make both types of borrowers better off and banks break even at the same time.

First, we have several observations for the case when $a = \underline{a}$. As I stated before, equation (2) implies that the first-best indifference curve of any type of borrower always passes through the point ($C = 1, r = 1$). At this point, the collateral rate is the same as the interest rate. The borrower's cost of debt is fixed at \$1. Therefore, the borrower has no incentive to increase or decrease the probability of success unless his actions are costly. It yields exactly the same utility as does the first-best contract. In the first best, with zero collateral, a bank's cost of \$1 should be covered entirely by debt interest because the market is perfectly competitive. It looks as if the borrower bears a fixed cost of debt at \$1.

Second, (ICM) implies that under risk-taking moral hazard, the borrowers might choose an action $a \geq \underline{a}$ if and only if $C \geq 1$. Now I show that any alternative contract set with $C \geq 1$ that can make both types of borrowers better off leads to a negative profit for a bank. For a chosen action a_i, the indifference curves of the type θ_i borrower have a slope of $-\dfrac{1 - a_i\theta_i}{a_i\theta_i}$. Thus, a higher action makes the indifference curves flatter and the bank's break-even lines even flatter (with $0 \leq \beta < 1$). In Figure 5.3 or Figure 5.4, the first-best indifference curve for any type of borrower has already crossed the bank's break-even line. Therefore, any alternative contract set beyond the point $C = r = 1$ cannot make both types of borrowers better off and yield a non-negative profit for a bank at the same time. Thus, the separating equilibrium of a contract set G for the good-type and B for the bad-type is unique.

5.2 Monopolistic Screening Equilibrium

I now consider monopoly. As I mentioned in Footnote 8 in Section 3, the equilibrium concept here is slightly different from that in Chan and Thakor (1987). To also consider the impacts of macro-economics conditions (such as risk-free rate) on the equilibrium results, Chan and Thakor (1987) assume that the deposit interest rate is endogenously determined so that a bank's equilibrium expected profit is always zero. But here I assume a fixed supply of deposit funding (at \$1 with zero risk-free rate) and a bank might earn a positive profit. This new definition allows me to study the bank's market power and compare it to that in the perfect competitive case. But the equilibrium contracts should be the same as those in Chan and Thakor (1987) if under the same setting.

The principal's program now becomes:

$$\max_{C_i, r_i} \Pi(C_i, r_i; a_i^*, \theta_i) = \lambda \cdot [a_B^* \theta_B \cdot r_B + (1 - a_B^* \theta_B) \cdot \beta C_B] + (1 - \lambda) \cdot [a_G^* \theta_G \cdot r_G + (1 - a_G^* \theta_G) \cdot \beta C_G] - 1,$$

$$\text{subject to } V(C_i, r_i; a_i^*, \theta_i) = L\theta_i - a_i^* \theta_i \cdot r_i - (1 - a_i^* \theta_i) \cdot C_i \geq 0,$$

$$\text{and } (ICM), (ICG) \text{ and } (ICB).$$

That is, to maximize the bank's expected profit given that the borrower who participates has a non-negative utility and subject to three incentive constraints. In the following analysis, I introduce each constraint step by step.

With full information and no moral hazard, a bank would never require the borrowers to secure the loan as collateral is costly ($\beta < 1$). Contrary to the competitive case, however, the good-type borrower is asked for a higher interest rate than the bad-type because the good-type borrower brings a higher surplus to the bank (with a higher expected return of $L\theta_i$) than the bad-type. In order to extract as much borrower surplus as possible, a bank, as a monopolist, charges the good-type borrower a higher interest rate.

With asymmetric information but no moral hazard, there are two potential zero-collateral equilibria: a pooling equilibrium or an equilibrium without the bad-type borrowers. Unlike the perfect competition case where the bad-type has incentive to mimic the good-type, in the pure monopoly case, it is the good-type who has incentive to mimic the bad-type because he is charged a higher interest rate. However, the good-type borrower still has a lower marginal cost to pledge higher collateral so the only way for the bad-type borrower to distinguish himself is to lower his collateral level. Nevertheless, the lowest collateral level is zero as in the full information case. In other words, collateral is not efficient as a sorting mechanism in this case. Therefore, the pooling contract attracts both types of borrowers but with a lower or even negative profit from the bad-type, whereas the good-type only contract extracts the most profit from the good-type borrower but the bad-type simply walks away. Which contract dominates in equilibrium depends on which one brings the highest profit to banks. But in any case, zero collateral is always optimal. At this point, my results are consistent with Besanko and Thakor (1987a).

With both asymmetric information and risk-taking moral hazard, a bank maximizes expected profits subject to the borrowers' participating constraints and (information and risk) incentive constraints. The additional incentive constraint is exactly the same as equation (ICM) that I showed in Section 5.1.2.

With zero collateral in equilibrium, $\underline{a}$ is again the optimal action. I also find that the pooling contract always yields a higher profit for a bank, it is the unique equilibrium.

Proposition 3 *In a purely monopolistic credit market with asymmetric information and risk-taking moral hazard, there is a unique pooling equilibrium with zero collateral and the same interest rate for both types* $r_i = \dfrac{L}{\underline{a}} - \dfrac{\gamma \underline{a}}{2[\lambda\theta_B + (1-\lambda)\theta_G]}$, *and the borrowers would always choose the optimal action of* $\underline{a}$.

To prove Proposition 3, I calculate the equilibrium profits for the pooling contract and the good-type only contract:

$$\Pi_P = [\lambda\theta_B + (1-\lambda)\theta_G] \cdot \underline{a} r_i - 1,$$

$$\Pi_G = (1-\lambda)(\underline{a}\theta_G r_G - 1) \text{ where } r_G = \frac{L}{\underline{a}} - \frac{\gamma \underline{a}}{2\theta_G}.$$

With Assumption $(A1c)$, $\Pi_P - \Pi_G = \lambda \cdot (L\theta_B - \frac{\gamma \underline{a}^2}{2} - 1) \geq 0$, so the pooling contract is always preferred by a bank in equilibrium.

SECTION 6

SHIRKING MORAL HAZARD

In this section, I consider shirking moral hazard (without risk-taking or adverse selection). The following set of assumptions is for this section only.

Assumption 3 (A3)

a. The action takes only one value and $D(a) = 0$.

b. $D(e)$ *is an increasing and convex function, and* $D'''(e) \geq 0$. *Moreover,* $D'(0) = 0$ *and* $D'(\overline{e}) = +\infty$.[1]

c. $p(e, \theta)$ *is linear in* e *with a zero intercept (so that we can write* $p(e, \theta) = ep_e$*).*

d. $p(e, \theta_1) > p(e, \theta_2)$ *for any* e *and for any* $\theta_1 > \theta_2$.

$(A3a)$ assumes away risk-taking moral hazard so that I can focus on shirking. $(A3b)$ guarantees an interior solution for e. Finally, $(A3c)$ and $(A3d)$ simplify the analysis. $(A3d)$ implies that the borrower's quality θ improves the return distribution in a First-order-Stochastic-Dominance (FSD) sense.[2] Most results in this section hold without these two assumptions except for Corollary 1 (see Section 6.1 for more details). A simple example that satisfies all assumptions in $(A1)$ and $(A3)$ is $p(e, \theta) = e\theta$.

6.1 Perfect Competition with Shirking

Now I consider the second-best equilibrium in a perfectly competitive credit market with shirking moral hazard only. As described in Section 3, perfect competition for loans among

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[1] This is similar to the Inada condition.

[2] That is, quality type θ and effort e are partially complementary in determining default risk. This assumption is consistent with Chan and Thakor (1987), but the opposite to that in Boot, Thakor, and Udell (1991) where they assume a substitutable relation. Since this paper focuses on moral hazard rather than adverse selection, this is not a key assumption.

banks produces credit contracts that maximize borrowers' expected utilities subject to two constraints (IR_{Π}) and (IR_V).

Faced with a credit contract (C, r), each borrower chooses an optimal effort level e^* that maximizes his expected utility:

$$e^* \in \arg\max_{e} V(C, r; e, \theta)$$

It can be rewritten with the following first-order condition:[3]

$$L\theta = D'(e^*) - p_e \cdot (C - r). \tag{IC_e}$$

I then study the impact of shirking moral hazard in equilibrium by solving the following program for the principal (let $p^* = p(e^*, \theta)$):

$$\max_{C,r} V(C, r; e^*, \theta) = e^* L\theta - p^* \cdot r - (1 - p^*) \cdot C - D(e^*), \tag{2'}$$

$$\text{subject to } (IR_{\Pi}), (IR_V) \text{ and } (IC_e).$$

This is a maximization problem with three endogenous variables. It needs additional assumptions to insure the existence and uniqueness of an optimal solution. In the following, I rewrite $(2')$ as a function of e and derive sufficient conditions for my equilibrium results.

Although the borrower chooses an effort level only after he and a bank both sign on the credit contract, it is mathematically more convenient to solve for the second-best effort first. Following prior literature of shirking moral hazard, I replace C and r by functions of e into the principal's objective function and solve for the optimal e.[4] Using (IC_e) and the binding (IR_{Π}) (as explained in Section 3), I denote $\phi^* = \beta + (1 - \beta) \cdot p^*$, and rewrite the equilibrium contract (C^*, r^*) as follows:

$$C^* = \frac{1 - e^*(L\theta - D'(e^*))}{\phi^*}, \tag{5}$$

$$r^* = \frac{1 + (\phi^*/p^* - 1) \cdot e^*(L\theta - D'(e^*))}{\phi^*}. \tag{6}$$

It is as if a bank has a rational expectation of how the borrower would react to a specific contract (C_j, r_j) by choosing an optimal effort level e_j^*, and therefore designs an optimal

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[3] I assume that e satisfies Assumption $(A1)$ ($\bar{e}$ is not too large) and $(A3)$ ($\bar{e}$ is not too small) to avoid corner solutions. Furthermore, Lemma 4 derives sufficient conditions for the "first-order approach" to be valid under my model setting. The reasoning is similar to Grossman and Hart (1983).

[4] In Chapter 5 of their book "The Theory of Incentives," Laffont and Martimort (2002) provide a method to solve the moral hazard model of "Two Outcomes with A Continuum of Effort Levels."

contract accordingly to mitigate the shirking problem. As a consequence, each feasible contract (C_j, r_j) has a one-to-one mapping to an optimal effort level e_j^* according to (IC_e). In addition, given e_j^*, each pair of C_j and r_j is also a one-to-one mapping to each other according to the binding (IR_Π).

In summary, the equilibrium is as if a bank maximizes the borrower's utility by picking an optimal effort level for the borrower. Let $p(e) = p(e,\theta)$ and $\phi(e) = \beta + (1-\beta)\cdot p(e)$, and the principal's program is reduced to a function of e:

$$\max_e V(e,\theta) = \frac{1}{\phi(e)} \cdot [eL\theta - 1 - (1-\phi(e))\cdot eD'(e)] - D(e). \tag{$2'_e$}$$

The first-order and second-order derivatives of $V(e,\theta)$ with respect to e are then given by:

$$\frac{dV(e,\theta)}{de} = \frac{1}{\phi^2(e)} \cdot [\beta\cdot(L\theta - D'(e)) + (1-\beta)\cdot p_e - \phi(e)(1-\phi(e))\cdot eD''(e)]. \tag{FOC_c}$$

$$\frac{d^2V(e,\theta)}{de^2} = -\frac{2(1-\beta)\cdot p_e}{\phi(e)} \cdot \frac{dV(e,\theta)}{de} - \frac{[2(1+\beta) - 3\phi(e)]\cdot D''(e) + (1-\phi(e))\cdot eD'''(e)}{\phi(e)}. \tag{SOC_c}$$

In order to use the "first-order approach" in discussing solutions of e, we need an additional assumption in the following set:

Assumption 4 (A4)

a. *Relaxed version:* $[2(1+\beta) - 3\phi(e)]\cdot D''(e) + (1-\phi(e))\cdot eD'''(e) \geq 0$ *for any* e.

b. *Strong version:* $\frac{1}{2} \leq \beta < 1$ *and* $D'(e)$ *is not too large relative to* $L\theta$ *so that we have* $\frac{d^2V(e,\theta)}{de^2} \leq 0$ *for any* e.

Lemma 4 *With Assumptions* $(A1)$, $(A3)$, *and* $(A4a)$, *the equilibrium effort under moral hazard must be a solution from the "first-order approach" (See proof in Appendix A).*

Grossman and Hart (1983) show that the "first-order approach" is valid if the CDFC (Concavity of distribution function condition) holds. Lemma 4 is similar to such a statement. The assumption sets $(A1)$, $(A3)$, and $(A4a)$ make sure that $\frac{d^2V(e,\theta)}{de^2} \leq 0$ for any e such that $\frac{dV(e,\theta)}{de} \geq 0$. So the optimal solution of e must be either at the corners or among the stationary points (where $\frac{dV(e,\theta)}{de} = 0$). Because $\frac{dV(e,\theta)}{de}|_{e=0} > 0$ and $\frac{dV(e,\theta)}{de}|_{e=\bar{e}} < 0$, we can rule out the corner solutions. As a result, the maximization problem of equation

$(2'_e)$ is tractable using the "first-order approach" with an extra step to compare several potential equilibria. In fact, with a stronger assumption $(A4b)$, the objective function is a hump shape with only one stationary point, so that we have Lemma 5.

Lemma 5 *With Assumptions* $(A1)$, $(A2)$, *and* $(A4b)$, *there is a unique solution of* e *that satisfies* $(FOC_c) = 0$ *(See proof in Appendix A).*

With Lemma 5, we can now use the "first-order approach" and rewrite equation (FOC_c) as:

$$L\theta = D'(e^{SB}) + \Gamma_c(e^{SB}, \theta, \beta), \tag{4^{SB}}$$

where

$$\Gamma_c(e^{SB}, \theta, \beta) = \frac{1}{\beta}[\phi(e^{SB})(1 - \phi(e^{SB})) \cdot e^{SB} D''(e^{SB}) - (1 - \beta) \cdot p_e]. \tag{NMC_c}$$

Traditional moral hazard models (See Footnote 4) discuss the second-best effort by comparing (4^{SB}) to (4^{FB}) in Section 4. In the second-best, in addition to the marginal benefit $L\theta$ and the disutility $D'(e^{SB})$, there is a third component - the net marginal cost of effort (called NMC hereafter). It is a function of e^{SB} and is affected by borrower type (θ), the cost of collateral (β) and the borrower's disutility function $(D(e))$. To facilitate the following analysis, I denote it as $\Gamma_c(e^{SB}, \theta, \beta)$ (see equation (NMC_c)).

In their Chapter 5, Laffont and Martimort (2002) show that NMC in their example is the marginal cost of effort that is due to the agent's limited-liability rents and is always positive. Because to reduce the limited-liability rents calls for decreasing effort, the second-best effort level is always below the first-best level.

Boot, Thakor, and Udell (1991) also study the competitive equilibrium with shirking. However, they show that when there is shirking moral hazard, borrowers still choose the first-best effort level. By allowing only two choices of effort level, they overlook the inefficiency in effort level because of shirking and costly collateral. In contrast, in my model, the first-best effort level can be achieved only with a specific set of parameters (see Lemma 6).

Lemma 6 *In perfect competition with shirking only, the first-best effort level is achieved if and only if* $\Gamma(e^{FB}, \theta, \beta) = 0$. *And in this case,* $C^{SB} = r^{SB} = \dfrac{1}{\phi(e^{FB})}$.

In my model, there are two components in NMC. The first component $\frac{1}{\beta}\phi(e^{SB})(1 - \phi(e^{SB})) \cdot e^{SB} D''(e^{SB})$ is the marginal cost of effort to reduce limited-liability rents (MCE).

The second component $\dfrac{(1-\beta)\cdot p_e}{\beta}$ is the marginal benefit of effort to increase the probability of success (MBE). If the combined effect is a net cost, the results in traditional moral hazard models remain the same that the second-best effort level is lower than the first-best. If it is a net benefit, the second-best effort level can be higher than the first-best. And that is Lemma 7.

Lemma 7 *With Lemma 5, the second-best effort level under shirking moral hazard can be higher or lower than the first-best level depending on the sign of the net marginal cost of effort* $\Gamma(e^{SB},\theta,\beta)$.

The following analysis focuses on how different frictions change the sign of Γ and thus determine the optimal effort level and contract under shirking moral hazard.

Clearly, if $\beta = 1$, $\Gamma = 0$. In this case, (4^{SB}) is exactly the same as (4^{FB}) and moral hazard is not an issue. If $0 \le \beta < \frac{1}{2}$, moral hazard has a comprehensive effect and there may be multiple stationary points (Lemma 4). Thus, in order to obtain intuitive properties, the following analysis is based on Assumption $(A4b)$ that collateral is costly but not too costly to sabotage the equilibrium (Lemma 5 with $\frac{1}{2} \le \beta < 1$).

Proposition 4 *In perfect competition with shirking only, there is a unique optimal solution of* $e^{SB} = e_1^*$ *that satisfies equation* (4^{SB}). *The equilibrium contract is then determined by equation* (5) *and* (6) *with* $e^* = e_1^*$.

- *If* $\Gamma(e^{FB},\theta,\beta) \le 0$ *so that* $\dfrac{dV(e,\theta)}{de}|_{e=e^{FB}} \ge 0$, *then the second-best effort* $e^{SB} \ge e^{FB}$, *with an optimal contract of* $C^{SB} \ge r^{SB} \ge 1$*;*
- *Otherwise,* $e^{SB} < e^{FB}$ *and* $0 < C^{SB} < r^{SB}$.

Proposition 4 is straightforward from Lemma 5 and Lemma 7. With the hump shape objective function of e, the second-best effort level is at the unique stationary point. That is, equation (4^{SB}) yields only one real-value solution e_1^* in the compact interval $[0,\overline{e}]$.

Corollary 1 *When* $\Gamma_c(e^{FB},\theta,\beta) < 0$, *the equilibrium collateral is higher than the equilibrium interest rate* $(C > r \ge 1)$. *Thus, it induces an effort level that is higher than the first-best level. Therefore, when* θ *that satisfies*

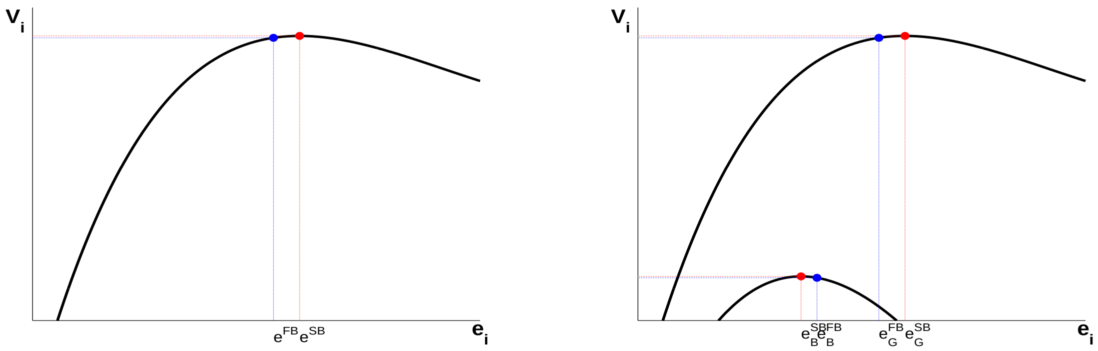
$$L\phi(e^{FB})\cdot(1-p(e^{FB})) - p_{e\theta} - (2\phi(e^{FB})-1)\cdot e^{FB}D''(e^{FB})p_\theta < 0,$$

the good borrower (with high θ) is more likely to be overcollateralized ($C > 1$) and to exert too much effort (compared to the first-best level) in equilibrium.

The condition of θ in Corollary 1 can be derived by taking the first-order derivative of Γ_c with respect to θ. With this condition, $\Gamma_c(e^{FB}, \theta_1, \beta) < \Gamma_c(e^{FB}, \theta_2, \beta)$ if $\theta_1 > \theta_2$. That is, the good borrower (with higher θ) has a lower NMC than the bad borrower. As a result, the good borrower is more likely to have a negative NMC that leads to overcollateralization and an above first-best effort level.

Figure 6.1 describes Corollary 1 with specifications of $p(e, \theta) = e\theta$ and $D(e) = 1/2\gamma e^2$ ($\gamma > 0$).[5] (a) gives an example where collateral yields a net marginal benefit of increasing effort above the first-best level ($\Gamma(e^{FB}, \theta, \beta) < 0$). For any given e in the picture, the borrower has a rational expectation of what credit contract a bank would offer in equilibrium. Thus, the borrower knows exactly where his expected utility ends up on the curve for each choice of effort level (as shown in (a)). There is a unique equilibrium (the red point on the right) where the borrower's expected utility is maximized. The blue point on the left provides a benchmark of what happen when there is shirking moral hazard and the borrower chooses the first-best effort level. It is lower than the second-best effort level but its corresponding expected utility is also lower, so it is not an equilibrium when there is shirking.

In picture (b), I show that the case in picture (a) is more likely to happen to a good-type borrower (with a higher θ), which is represented by the higher hump shape in (b). The



(a) $\Gamma(e^{FB}, \theta, \beta) < 0$ (b) Two Types

Figure 6.1: Corollary 1

[5]The parameter set that I use to simulate Figure 6.1 is: $L = 4.5$, $\gamma = 5$, $\beta = 0.7$, $\theta_G = 1$, $\theta_B = 0.8$, and $\overline{a} = \underline{a} = 1$.

bad-type borrower is more likely to end up in the smaller hump shape below with an opposite situation: the equilibrium effort level is below the first-best.

Corollary 2 *In the second-best equilibrium, banks always use positive collateral. Higher collateral induces higher effort level.*

Under shirking moral hazard, if there is no collateral, the borrower tends to exert an effort level that is lower than the first-best because effort is costly. (IC_e) constraint implies that higher collateral induces higher effort level. Therefore, in equilibrium, it is optimal for a bank to require positive collateral to encourage a higher effort level. This result is different from that in Boot, Thakor, and Udell (1991) where the equilibrium contract for the good borrower is unsecured when there is shirking. In their model, the good-type borrower exerts a first-best effort level even when there is shirking because they impose an exogenous boundary on the choices of available effort levels. Thus, the "first-best" effort level in their model is in fact the relatively better choice of the two choices that are available to the borrower, but not the "true" first-best effort level within the choices of a continuum of effort level. In other words, their results are corner solutions in my model when choices of effort level are bounded.[6]

Corollary 3 *In the second-best equilibrium, all the inefficiencies come from the assumption of costly collateral ($\beta < 1$). A lower cost of collateral (high β) improves social efficiency, but the relation between β and effort (or collateral) is not monotonic.*

In their Proposition 3, Boot, Thakor, and Udell (1991) claim that an increase in β reduces the collateral requirement for the bad borrower.[7] This is consistent with Lemma 6 when the first-best effort level is achieved with a specific set of parameters. Unlike Boot, Thakor, and Udell (1991), with an endogenously determined second-best effort level in my model, the relation between β and collateral is not always monotonic.

[6] Although I do not include my results here, I also extend my model to the cases when effort level has a lower bound and (or) a upper bound. In those cases, the first-best equilibrium can sometimes be achieved when the borrower has a very high productivity. This extension can explain Boot, Thakor, and Udell (1991)'s results. In addition, when the borrower's productivity is very low, collateral sometimes might not be effective enough to encourage effort and zero collateral is used under second best.

[7] In their model, they assume the borrower's effort level is bounded so that the good borrower gets the first-best contract and exerts first-best effort even with shirking moral hazard. Although the bad borrower is offered a secured contract instead under the second-best, he still choose a first-best effort level due to the assumption of limited choices of effort level.

The economic intuition of Corollary 3 is as follows. If collateral is costless ($\beta = 1$), full collateral is always used.[8] In this case, collateral provides a punishment mechanism for a bank to eliminate the limited-liability rents. With $C = r = 1$, the borrower pays 1 no matter what and collateral provides enough incentive to induce a first-best effort level. Therefore, moral hazard is no longer an issue and costless collateral imposes no inefficiency.

However, in this thesis, I assume that collateral is costly. This assumption of $\beta < 1$ has two effects. The first effect is to weaken the effectiveness of collateral as a punishment mechanism to induce a higher effort level in order to reduce the limited-liability rents. As a consequence, a higher collateral level might be used to reduce the inefficiency from the limited-liability rents. But because collateral is itself costly, higher collateral also introduces extra inefficiency. In addition, $\beta < 1$ has an indirect effect to induce higher effort because the borrower knows that if the project fails, he is the one who bears the cost of collateral. In summary, given $\beta < 1$, banks and the borrower face a trade-off between reducing inefficiency from the limited-liability rents and reducing inefficiency from the cost of collateral itself.

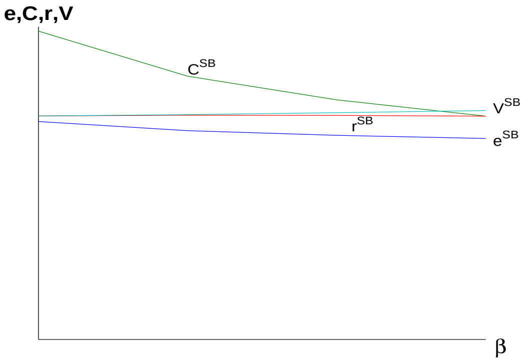
Clearly, from $(2'_e)$, a higher β leads to a higher borrower value and thus introduces less inefficiency.[9] However, because collateral can reduce the limited-liability rents but is costly, a lower collateral level does not necessarily improve efficiency. Thus, the relation between β and equilibrium collateral might not be monotonic. Because of the monotonic relation between effort and collateral, we can obtain the same non-monotonic relation between β and the equilibrium effort level.

An alternative way to demonstrate Corollary 3 is to consider the trade-off between MCE and MBE. Because a larger β decreases MBE and MCE at the same time, it is not clear ex-ante which marginal effect dominates in determining the optimal effort level and collateral level.

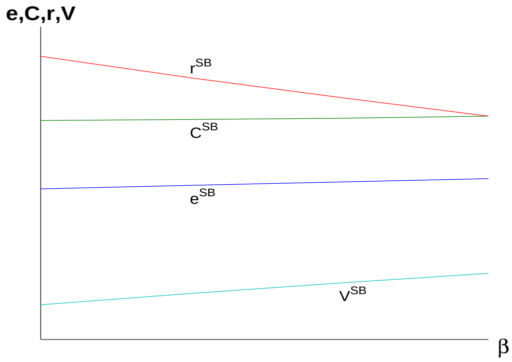
In Figure 6.2, I show how the equilibrium effort level e, collateral C, interest rate r and borrower's value V change with β. I use the same example as in Figure 6.1. Picture (a) is corresponding to Figure 6.1(a), and (b) is to the lower hump shape in Figure 6.1(b). In picture (a), the equilibrium effort and collateral both decrease with β, where as in picture (b), they both increase with β. In both pictures, the borrower's value increases as

[8] Chan and Thakor (1987) also discuss this case in their model.

[9] $(2'_e)$ can be rewritten as $V(e,\theta) = eD'(e) - D(e) - \dfrac{1 - e(L\theta - D'(e))}{\phi(e)}$. Therefore, $C^* > 0$ and equation (5) imply that a higher β increases borrower value.



(a) $\Gamma(e^{FB}, \theta, \beta) < 0$



(b) $\Gamma(e^{FB}, \theta, \beta) > 0$

Figure 6.2: Corollary 3

β increases, because a lower cost of collateral improves social welfare.

6.2 Pure Monopoly with Shirking

In this subsection, the second-best equilibrium in a monopolistic credit market with shirking moral hazard is considered. When a bank has monopoly power, his program generates credit contracts that maximize his expected profit subject to the two constraints (IR_Π) and (IR_V) and an incentive constraint of effort. That is:

$$\max_{C,r} \Pi(C, r; e^*, \theta) = p^* \cdot r + (1 - p^*) \cdot \beta C - 1, \tag{$1'$}$$

$$\text{subject to } (IR_\Pi), (IR_V) \text{ and } (IC_e).$$

In this case, it is the (IR_V) constraint that always binds. Now that the bank has market power, it maximizes its profits and get the entire total surplus. That means the borrower gets a zero utility in equilibrium, which is just enough to get him to sign the credit contract.

Following the same method as in Section 6.1, I solve the principal's program in the pure monopoly case. With (IC_e) and the binding (IR_V), there is a one-to-one mapping from the optimal effort level e_i^* to C_i^* and to r_i^*. Therefore, the equilibrium contract (C^*, r^*) can be rewritten as:

$$C^* = e^* D'(e^*) - D(e^*), \tag{$5'$}$$

$$r^* = \frac{e^* L\theta - (1 - p^*) \cdot e^* D'(e^*) - p^* \cdot D(e^*)}{p^*} = C^* + \frac{e^*(L\theta - D'(e^*))}{p^*}. \tag{$6'$}$$

Similarly, the principal's program is reduced to a function of e:

$$\max_{e} \Pi(e, \theta) = eL\theta - 1 - (1 - \phi(e)) \cdot eD'(e) - \phi(e)D(e), \tag{$1'_e$}$$

Then I calculate the first-order and second-order derivatives of $\Pi(e,\theta)$ with respect to e as follows:

$$\frac{d\Pi(e,\theta)}{de} = L\theta - D'(e) + (1-\beta)p_e \cdot [eD'(e) - D(e)] - (1-\phi(e)) \cdot eD''(e). \qquad (FOC_m)$$

$$\frac{d^2\Pi(e,\theta)}{de^2} = -[2(1+\beta) - 3\phi(e)] \cdot D''(e) - (1-\phi(e)) \cdot eD'''(e). \qquad (SOC_m)$$

With (FOC_m) and (SOC_m), it can be shown that Lemma 4 and Lemma 5 still hold under the pure monopoly case. Moreover, Assumption $(A4b)$ in Lemma 5 can be relaxed to just $\frac{1}{2} \leq \beta < 1$. Again, I focus on the solutions from the "first-order approach" in Lemma 5 and rewrite equation (FOC_m) as:

$$L\theta = D'(e^{SB}) + \Gamma_m(e^{SB},\theta,\beta), \qquad (4'^{SB})$$

where

$$\Gamma_m(e^{SB}|\theta,\beta) = (1-\phi(e^{SB})) \cdot e^{SB}D''(e^{SB}) - (1-\beta)p_e \cdot [e^{SB}D'(e^{SB}) - D(e^{SB})]. \ (NMC_m)$$

Lemma 8 *In pure monopoly with shirking only, the first-best effort level is achieved if and only if* $\Gamma_m(e^{FB},\theta,\beta) = 0$. *And in this case,* $C^{SB} = r^{SB} = e^{FB}L\theta - D(e^{FB}) \geq \frac{1}{\phi(e^{FB})}$.

The reasoning is again similar to that in Section 6.1. Hence, Lemma 8 is similar to Lemma 6 but with a different formula of NMC. Proposition 4 is also similar under pure monopoly but with (NMC_m) instead of (NMC_c). Chan and Thakor (1987) show that higher collateral induces higher effort level under pure monopoly with $\beta = 1$. My model extends their results to the cases when collateral is costly.

Lemma 9 *With the same set of parameters, in order to induce a given effort level, higher collateral is needed under pure monopoly than under perfect competition.*

Lemma 9 can be derived by the equilibrium results of collateral under the two notions of competition with (IR_V) or (IR_Π). Equation (5) and (5′) suggest that for the same e, $C^*_m - C^*_c = S^*_c \geq 0$. Therefore, to induce a given effort level, higher collateral is needed under pure monopoly than under perfect competition.

Lemma 10 *With the same set of parameters, the second-best equilibrium effort level under pure monopoly is greater than that under perfect competition.*

Lemma 10 can be derived by comparing (FOC_m) to (FOC_c). Suppose e_m^* is the second-best effort level under pure monopoly such that $(FOC_m) = 0$. I can show that with $e = e_m^*$, $(FOC_c) = (1-\beta)p_e\phi(e_m^*)\cdot(C_c^*(e_m^*) - C_m^*(e_m^*)) < 0$ (see Lemma 9). Therefore, in order to get $(FOC_c) = 0$, the second-best effort level under perfect competition e_c^* must be lower than e_m^*.

Proposition 5 *In a credit market under shirking moral hazard, all else equal, competition improves market efficiency.*

From equation (3) in Section 3, the total surplus in perfect competition is given by:

$$S_c^* = \frac{1}{\phi_c^*}[e_c^* L\theta - 1 - (1-\phi_c^*)\cdot e_c^* D'(e_c^*)] - D(e_c^*).$$

Whereas the total surplus in pure monopoly is given by:

$$S_m^* = e_m^* L\theta - 1 - (1-\phi_m^*)\cdot e_m^* D'(e_m^*) - \phi_m^* D(e_m^*).$$

When $\beta = 1$ so that $\phi^* = 1$, the first-best effort level can be achieved under both market structures so that both equilibria yield the same total surplus. In this case, monopoly power does not matter to social welfare. However, when collateral is costly ($\beta < 1$), with the same e_m^*, $S_m^*(e_m^*) = \phi(e_m^*)\cdot S_c^*(e_m^*) < S_c^*(e_m^*)$. In addition, because e_c^* maximizes S_c, we also have $S_c^*(e_m^*) < S_c^*(e_c^*)$. Therefore, $S_m^*(e_m^*) < S_c^*(e_c^*)$ and competition improves market efficiency.

Figure 6.3 illustrates Lemma 10 and Proposition 5.[10] I use the same set of parameters as in Figure 6.2(b). Under pure monopoly, the second-best effort level is higher than that under perfect competition, but the total surplus is smaller.

[10]The parameter set that I use to simulate Figure 6.3 is: $L = 4.5$, $\gamma = 5$, $\beta = 0.7$, $\theta = 0.8$, and $\overline{a} = \underline{a} = 1$.

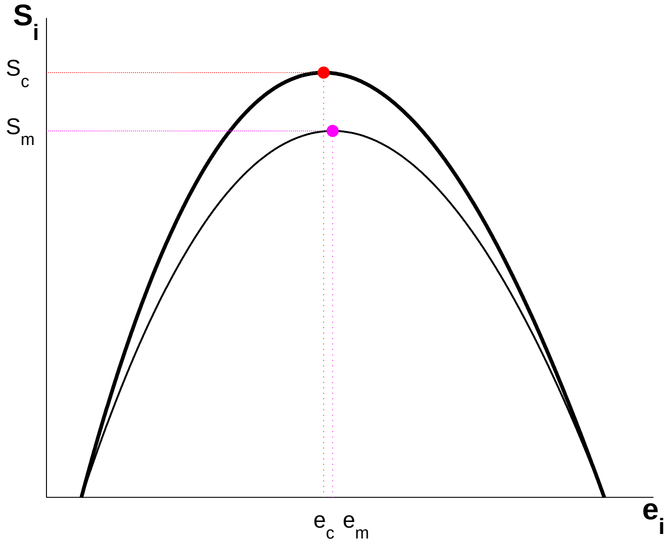


Figure 6.3: Proposition 5

SECTION 7

TWO-DIMENSIONAL MORAL HAZARD

In the last two sections, I have discussed the equilibrium results with each type of moral hazard, risk-taking and shirking, separately. In this section, I study the equilibria with two-dimensional moral hazard in a single model. Because I do not find any new insight in the pure monopoly case other than what I have shown in Section 6.2, I only focus on the perfect competition case in this section.

For tractability, I impose the following set of assumptions for this section:

Assumption 5 (A5)

a. $\overline{a} > \underline{a} > 0$ *and* $D(a) = 0$ *for all* a.

b. The disutility of effort function $D(e)$ *is increasing and convex in* e *(*$D'(e) > 0$ *and* $D''(e) \geq 0$*), and* $D'''(e) \geq 0$*. Moreover,* $D'(0) = 0$ *and* $D'(\overline{e}) = +\infty$*.*[1]

c. $p(e, a, \theta)$ *is linear in* e *with a zero intercept (so that we can write* $p(e, a, \theta) = ep_e(a, \theta)$*).*

d. $p(e, a, \theta_1) > p(e, a, \theta_2)$ *for any* $\theta_1 > \theta_2$*.*

Assumption $(A5)$ is similar to Assumption $(A3)$ but with the risk-taking variable a. $(A5b-d)$ are very similar to $(A3b-d)$. In Assumption $(A5a)$, I set $D(a)$ to be zero for two reasons. First, as I have shown in Section 5, by setting $D(a) = 0$, the model captures the pure risk-taking effect. My main results of zero collateral and highest risk-taking in equilibrium in Section 5 hold with or without the cost of risk-taking. Similarly here, if $D(a)$ is small relative to $C-r$ (or even negative), the cost (or benefit) of risk-taking is just a second-order effect and does not affect my main results (also see the analysis of Figure 7.1).[2] Second,

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[1]This is similar to the Inada condition.

[2]Intuitively, a positive cost of risk-taking suggests that it is costly to find a less risky project. Suppose gambling in Las Vages is always a costless option, it makes sense to assume $D(a) > 0$. However, assuming $D(a) < 0$ is equivalent to assume that the borrower is risk-averse. While both cases are interesting in practice, in this thesis, I focus on the case $D(a) = 0$ to capture the pure risk-taking effect and set up a benchmark.

if $D(a)$ is large enough, the equilibrium effort, risk-taking, collateral and interest rate all change simultaneously, and it is not clear ex-ante which incentive constraint binds ((IC_e) or (IC_a)) in equilibrium. In order to make the model tractable, I assume $D(a) = 0$ to simplify the analysis.

Faced with a credit contract (C, r), each borrower chooses a set of optimal actions e^* and a^* to maximize his expected utility:

$$\{e^*, a^*\} \in \arg\max_{e,a} V(C, r; e, a, \theta) \tag{IC}$$

For any given a^*, the incentive constraint of e now becomes:

$$L\theta = D'(e^*) - p_e(a^*, \theta) \cdot (C - r). \tag{IC_e}$$

By assumption, risk-control a does not affect expected return of the project and has no direct cost. In equilibrium, it yields corner solutions only (either $\overline{a}$ or $\underline{a}$), depending on the comparison between C and r:

$$a^* = \underline{a} \text{ if } C < r \text{ and } a^* = \overline{a} \text{ otherwise.} \tag{IC_a}$$

Similarly, I then study the impact of the two-dimensional moral hazard in equilibrium by solving the following program for the principal (let $p^* = p(e^*, a^*, \theta)$):

$$\max_{C,r} V(C, r; e^*, a^*, \theta) = e^* L\theta - p^* \cdot r - (1 - p^*) \cdot C - D(e^*), \tag{$2'$}$$

$$\text{subject to } (IR_\Pi), (IR_V), (IC_e) \text{ and } (IC_a)$$

Following the same methodology as in Section 6.1, let $\phi^* = \beta + (1 - \beta) \cdot p^*$, and the equilibrium contract (C^*, r^*) remains the same as in equation (5) and (6). Therefore, the equilibrium is as if a bank is maximizing the borrower's utility by picking an optimal set of actions for the borrower. Let $p(e) = p(e, a^*, \theta)$ and $\phi(e) = \phi(e) = \beta + (1 - \beta) \cdot p(e)$, and the principal's program is reduced to:

$$\max_{e} V(e, a^*, \theta) = \frac{1}{\phi(e)} \cdot [eL\theta - 1 - (1 - \phi(e)) \cdot eD'(e)] - D(e), \tag{$2'_e$}$$

$$\text{subject to } a^* = \underline{a} \text{ if } L\theta < D'(e) \text{ and } a^* = \overline{a} \text{ otherwise.} \tag{IC'_a}$$

For each optimal a^*, the first-order and second-order derivatives of $V(e, \theta)$ with respect to e are given by:

$$\frac{\partial V(e, a^*, \theta)}{\partial e} = \frac{1}{\phi^2(e)} \cdot [\beta \cdot (L\theta - D'(e)) + (1 - \beta) \cdot p_e(a^*, \theta) - \phi(e)(1 - \phi(e)) \cdot eD''(e)]. \tag{FOC}$$

$$\frac{\partial^2 V(e,a^*,\theta)}{\partial e^2} = -\frac{2(1-\beta)\cdot p_e(a^*,\theta)}{\phi(e)}\cdot\frac{\partial V(e,a^*,\theta)}{\partial e} - \frac{[2(1+\beta)-3\phi(e)]\cdot D''(e)+(1-\phi(e))\cdot eD'''(e)}{\phi(e)}. \quad (SOC)$$

In order to use the "first-order approach" in discussing solutions of e, we also need an additional set of assumptions that is very similar to Assumption $(A4)$ but with $a = a^*$:

Assumption 6 (A6)

a. *Relaxed version: For each optimal* a^*, $[2(1+\beta)-3\phi(e)]\cdot D''(e)+(1-\phi(e))\cdot eD'''(e) \geq 0$ *for any* e.

b. *Strong version: For each optimal* a^*, $\frac{1}{2} \leq \beta < 1$ *and* $D'(e)$ *is not too large relative to* $L\theta$ *so that we have* $\frac{\partial^2 V(e,a^*,\theta)}{\partial e^2} \leq 0$ *for any* e.

Lemma 11 *With Assumptions* $(A1)$, $(A2)$ *and* $(A6a)$, *the equilibrium effort under moral hazard must be either* e^{FB} *or a solution from the "first-order approach" (See proof in Appendix A).*

This is, again, similar to Grossman and Hart (1983)'s CDFC statement of the validity of the "first-order approach". However, with two possible actions $\overline{a}$ and $\underline{a}$ in equilibrium, the objective function here does not fulfill the strict concavity condition. So Lemma 11 is also a little bit different from Lemma 4. With Assumption $(A6a)$, now the maximization problem can be solved piecewisely by dividing the objective function into two regions (when $L\theta < D'(e)$ and when $L\theta \geq D'(e)$). In other words, the maximization problem remains tractable using the "first-order approach" with an extra step to compare several potential equilibria. In fact, with the stronger assumption $(A6b)$ (it is exactly the same as $(A4b)$), the objective function is a hump shape (or a part of the hump shape) in each region, so that we have Lemma 12.

Lemma 12 *With Assumptions* $(A1)$, $(A2)$ *and* $(A6b)$, *there are at most two potential solutions of* e *under moral hazard: the smaller one satisfies* $(FOC) = 0$ *with* $a = \underline{a}$, *and the larger one is either* e^{FB} *(if with* $e = e^{FB}$ *and* $a = \overline{a}$, $(FOC) < 0$*) or the one that satisfies* $(FOC) = 0$ *with* $a = \overline{a}$ *(See proof in Appendix A).*

Again, I focus on the solutions from the "first-order approach" in Lemma 12 and rewrite equation (FOC) as:

$$L\theta = D'(e^{SB}) + \Gamma(e^{SB}, a^{SB}, \theta, \beta), \tag{4^{SB}}$$

where

$$\Gamma(e^{SB}, a^{SB}, \theta, \beta) = \frac{1}{\beta}[\phi(e^{SB})(1 - \phi(e^{SB})) \cdot e^{SB} D''(e^{SB}) - (1 - \beta) \cdot p_e(a^{SB}, \theta)]. \tag{NMC}$$

Similarly, there are two components in (NMC): MCE and MBE. Whether the net effect is benefit or cost now depends on the interaction among different frictions: cost of effort (shirking, e), limited liability (risk-taking, a), and cost of collateral (β), given borrower's quality type (θ). Therefore, Lemma 7 still holds with $a = a^*$. With two potential a^* in equilibrium, we have Proposition 6.

Proposition 6 *In a competitive equilibrium under two-dimensional moral hazard (shirking and risk-taking: $\overline{a} > \underline{a}$), there are three potential equilibria:*

(Equilibrium 1:) $a^{SB} = \underline{a}$, *and* $e^{SB} = \underline{e_2^*}$ *that satisfies equation* (4^{SB}) *with* $a = \underline{a}$.
In this case, $e^{SB} < e^{FB}$ *and* $0 < C^{SB} < r^{SB}$.

(Equilibrium 2:) $a^{SB} = \overline{a}$, *and* $e^{SB} = e^{FB}$ *if* $(FOC) < 0$ *with* $e = e^{FB}$ *and* $a^* = \overline{a}$.
In this case, $C^{SB} = r^{SB} \geq \frac{1}{\phi(e^{FB})}$.

(Equilibrium 3:) $a^{SB} = \overline{a}$, *and* $e^{SB} = \overline{e_2^*}$ *that satisfies equation* (4^{SB}) *with* $a = \overline{a}$.
In this case, $e^{SB} \geq e^{FB}$ *and* $C^{SB} \geq r^{SB} \geq 1$.

Which equilibrium dominates depends on which one yields the highest $V(e^{SB}, a^{SB}, \theta)$ *based on equation* $(2_e')$. *The equilibrium contract is then determined by equation* (5) *and* (6).

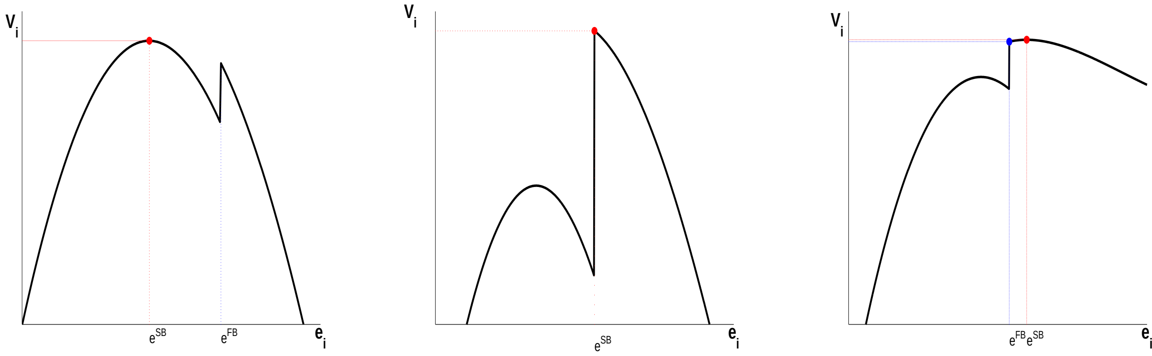
Proposition 6 can also be derived from Lemma 12. It considers risk-taking moral hazard as well as shirking by allowing $\overline{a} > \underline{a}$. In this case, the objective function $(2_e')$ consists of two hump shapes (the one on the right might be only the right part of a hump shape). Thus, there are only two peak values of e that lead to two potential equilibria: one is Equilibrium 1 in Proposition 6, and the other is either Equilibrium 2 or Equilibrium 3. One of them usually yields a higher value for the borrower than the other and is the real equilibrium. However, in a special case where the two equilibria yield the same value for the borrower, two equilibria can both exist.

Figure 7.1 gives examples of the three equilibria that are described in Proposition 6.[3] In (a), collateral yields a net marginal cost of increasing effort $(\Gamma(e^{SB}, \underline{a}, \theta, \beta) > 0)$ so that the second-best effort level is lower than the first-best effort level. In this case, a bank chooses to use a lower level of collateral $(C^{SB} < r^{SB})$ because collateral is not very effective and the borrower always takes the most risk $(a^{SB} = \underline{a})$.

Picture (b) is the case when risk-taking has a large impact on the project. In this case, the first-best effort level can be achieved. Collateral is also very effective so that a bank uses collateral the same rate as interest rate at $1/\phi(e^{FB})$. As a result, collateral also has a side effect on controlling risk-taking and the borrower chooses the lowest risk-taking $(a^{SB} = \overline{a})$ in equilibrium.

Picture (c) is quite similar to Figure 6.1 but with $\overline{a} > \underline{a} > 0$. In this case, collateral yields a net marginal benefit of increasing effort $(\Gamma(e^{SB}, \underline{a}, \theta, \beta) < 0)$ so that the second-best effort level is higher than the first-best effort level. Therefore, collateral is more effective than that in (a) and a bank chooses to use higher collateral $(C^{SB} > r^{SB})$. Again, as a side effect, higher collateral also induces the lowest risk-taking of borrower in equilibrium.

The discontinuity points in each picture are due to the extreme choices of risk-taking a. These points happen to be at $e = e^{FB}$ because I set $D(a) = 0$ (see equation (IC'_a)). However, if the cost of risk-taking is large enough, there are several second-order effects. First of all, each discontinuity point moves to the right where $e > e^{FB}$ because now a higher



(a) $e^{SB} < e^{FB}$ and $a^{SB} = \underline{a}$ (b) $e^{SB} = e^{FB}$ and $a^{SB} = \overline{a}$ (c) $e^{SB} > e^{FB}$ and $a^{SB} = \overline{a}$

Figure 7.1: Proposition 6

[3]The parameter sets that I use to simulate these pictures are as follows. For (a): $L = 5$, $\gamma = 5$, $\beta = 0.7$, $\theta = 0.7$, $\overline{a} = 1$, and $\underline{a} = 0.95$; for (b): $L = 4.5$, $\gamma = 5$, $\beta = 0.7$, $\theta = 0.8$, $\overline{a} = 1$, and $\underline{a} = 0.5$; for (c): $L = 4.5$, $\gamma = 5$, $\beta = 0.7$, $\theta = 1$, $\overline{a} = 1$, and $\underline{a} = 0.5$.

collateral is needed to switch the borrower's choice of risk-taking. However, collateral is still a side effect because a low a is still chosen when the collateral rate is lower than the interest rate, as shown on the left part of the picture. Second, the borrower value at each point is lower than before because of the cost of risk-taking. Last but not least, the gap in value at each discontinuity point is smaller. In fact, when the cost of risk-taking is very large so that the gap becomes negative, the borrower always chooses the highest risk-taking in equilibrium no matter how much collateral is pledged.

Corollary 4 *In equilibrium, collateral is used by banks only to encourage higher effort but not for the purpose of inhibiting risk-taking. But when a bank uses high collateral ($C > r \geq 1$), collateral has a side effect on lowering risk-taking.*

Corollary 4 is straight forward in Equilibrium 1 where $0 < C^{SB} < r^{SB}$, because the borrower chooses $a^{SB} = \underline{a}$ even with a positive collateral level. In Equilibrium 2 or Equilibrium 3 where $C^{SB} \geq r^{SB} \geq 1$, although the borrower chooses less risk-taking ($a^{SB} = \overline{a}$), it is only a side effect of collateral that is due to the positive impact of lower risk-taking on the effectiveness of collateral in encouraging higher effort. (4^{SB}) implies that, with $\frac{1}{2} \leq \beta < 1$, lower risk-taking leads to a lower MCE and a higher MBE, all else equal. That is, lower risk-taking increases the marginal effect of increasing effort. Therefore, it strengthens the function of collateral on encouraging higher effort.

However, lower risk-taking also make collateral more effective through ϕ^*, which in turn decreases the borrower's value (see equation ($2'_e$)). Therefore, it is not clear, ex-ante, whether banks choose to use higher collateral ($C^{SB} \geq r^{SB} \geq 1$). In equilibrium, either equilibrium in Proposition 6 might exist depending on different sets of parameters.

SECTION 8

DISCUSSION AND CONCLUSION

In this thesis, I re-examine the incentive function of collateral in a credit contract with different market frictions, including costly effort (shirking), limited liability (risk-taking), costly collateral, imperfect competition, and adverse selection. I show that when collateral is costly, the optimal level of collateral in equilibrium depends on the interactions of these market frictions. Particularly, banks use collateral to encourage higher effort from borrowers. However, the cost of collateral might bring inefficiency to both the second-best effort level and collateral level. As a result, when collateral is more effective in reducing limited-liability rents and in inducing higher effort, higher collateral (than the equilibrium interest rate) is used in equilibrium which leads to an effort level that is above the first-best. In this case, collateral also has a side effect of inhibiting risk-taking. However, when collateral is not as effective, lower collateral level (than the equilibrium interest rate) is used in equilibrium with an effort level that is below the first-best and collateral has no impact on risk-taking. This is a surprising result compared to conventional wisdom.

In Section 8.1, I discuss how my model can help explain the mixed results in the empirical studies. In addition, I show that while the interaction of adverse selection and competition reduces social efficiency, the interaction of shirking moral hazard and competition might improve social efficiency. These opposite predictions deserve further investigations.

In Section 8.2, I propose two testable hypotheses for future empirical research.

Finally, in Section 8.3, I discuss a few limitations of the model, and then propose potential extensions for future research in theory.

8.1 Empirical Implications

The first empirical implication is regarding the relation between the amount of collateral and borrower's type. Existing theories carefully elaborate their predictions based on certain assumptions. However, some empirical papers interpret these predictions differently and present contradicted results. For example, Boot, Thakor, and Udell (1991) gives a simple example with only two choices of effort levels and predicts an equilibrium with secured bad-type and unsecured good-type contracts. But at the end of their paper, they also emphasize that this result is based on the assumptions that effort and quality are substitutes and that the moral hazard problem is moderate. They even state explicitly that if the first assumption changes that effort and quality are complementaries, the results may be exactly the opposite. However, Jimenez, Salas, and Saurina (2006) cite Boot, Thakor, and Udell (1991) to develop their Hypothesis 1 directly without controlling for these two assumptions. In addition, they use a dummy variable as a proxy for collateral, whereas the economic intuition of collateral in the model is the ratio of the collateral amount over loan size. They then present a positive relation between the collateral dummy and the borrower's credit quality and attribute it to the screening function of collateral under adverse selection.

Steijvers and Voordeckers (2009) review empirical research on the function of collateral and add that other papers (e.g., Chakraborty and Hu (2006)) find an opposite relation to that in Jimenez, Salas, and Saurina (2006) and explain it with the incentive function of collateral to mitigate the moral hazard problem. This is consistent with conventional wisdom that the bad-type borrowers have higher risk than the good-types so that a bank requires higher collateral from the bad-types to control their risks. In contrast, I show that whether a borrower pledges a higher or lower collateral level depends on more than one conditions. My model implies that, in order to retrieve the real relation between collateral amount and borrower's quality type, one needs to control for different frictions. For example, as I have shown in Proposition 4, how much collateral to pledge in equilibrium depends on not only the borrower's quality type (θ) but also other factors such as the complementary or substitutable relation between effort and quality in determining default risk and the cost of collateral (β).[1] That is, at least two more variables should be included in the empirical test: the cost of collateral, and the interaction between effort and quality in determining default risk.[2]

Another empirical implication is about the proxy for risk. Boot, Thakor, and Udell (1991) has a discussion of the two measures of risk: default risk and project variance.

[1]Empirically, β can be affected by many factors. For example, a better lending relationship might reduce information asymmetry of the borrower's assets and enable banks to pick an asset for collateral with a higher β (e.g., Harhoff and Korting (1998), Steijvers and Voordeckers (2009)).

[2]For example, one way to control for the interaction between effort and quality empirically is to add in an industry fixed effect: In an industry where quality has a large impact on controlling risk-taking (like manufacture firms), it is more likely to be a complementary relation; whereas in an industry where risk-taking is almost irrelevant to firm quality (like pharmaceutical companies), it might be a substitute relation or no relation.

However, they do not provide a clear link between different variables in the model and the empirical proxies. Existing theories do not have a consistent presentation of risk in the models of credit contracts. For example, Stiglitz and Weiss (1981) use the same variable θ for type as well as for risk.[3] In their model, the borrower's type is reflected by the average risk of their project with a Mean-Preserving-Spread (MPS) technology. In Chan and Thakor (1987) and in Boot, Thakor, and Udell (1991), whether type θ and effort e are monotonously related to risk depends on which risk measure to use. A higher θ or (and) a higher e reduce default risk in a First-order-Stochastic-Dominance (FSD) sense so that their impacts on project variance depend on how bad the borrower is and how severe the shirking problem is. Additionally, the equilibrium default risk is determined not only by the borrower's quality type but also by the equilibrium effort level that is endogenously determined. However, because Boot, Thakor, and Udell (1991) assume that effort and quality are substitutes, the equilibrium default risk might not move in the same direction as does borrower type. Consequently, as Jimenez, Salas, and Saurina (2006) test the relation between credit quality (as a proxy for default risk) and collateral dummy in their Hypothesis 1, they do not precisely test the prediction of Boot, Thakor, and Udell (1991). In addition, other papers (e.g., Berger and Udell (1995), John, Lynch, and Puri (2003)) use loan interest rate as a proxy for borrower risk which makes the literature even more confusing.[4] Therefore, in order to construct a precise hypothesis, more theoretical guidance is needed.

This thesis also contributes to the literature by defining an unambiguous variable of risk and establish a clear link between variables in the model and the empirical proxies. I create a separate risk-control variable a. As I show in Table 8.1, all else equal, a lower a implies a higher risk-taking, no matter whether it is measured by default risk or project variance. Moreover, I assume that actions (both effort e and risk-control a) and type are complementary. In equilibrium, if collateral is moderately costly, a good-type borrower exerts a high effort level and take a high action so that type and equilibrium default risk

[3]Berger and Udell (1990) also do not distinguish between borrower type and loan risk. In fact, they use credit risk as a proxy for loan risk.

[4]Berger and Udell (1990) use excess loan rates as a proxy for loan risk. Whereas Petersen and Rajan (1995) use loan rate as a proxy for a bank's monopoly power. Berger and Udell (1995) use loan rate as a proxy for borrower risk and show that it has no statistical significant relation with collateral. However, John, Lynch, and Puri (2003) find a positive relation and attribute it to the imperfection in the credit rating process and the effect of collateral on the agency cost of equity (manager's perk consumption), which are not considered in my model. They do control for credit rating but not for the cost of collateral.

Table 8.1: Models of the Two-Dimensional Moral Hazard

Models	**Shirking** (e)	**Risk-taking** (a)	**Both** (e **and** a)
$\tilde{\mathbf{R}}$ **Technology**	FSD of e	MPS of a	both
Success:	L with prob. $e\theta$	$\frac{L}{a}$ with prob. $a\theta$	$\frac{L}{a}$ with prob. $ea\theta$
Failure:	0 with prob. $1 - e\theta$	0 with prob. $1 - a\theta$	0 with prob. $1 - ea\theta$
$\mathbf{E}(\tilde{\mathbf{R}})$	$eL\theta$	$L\theta$	$eL\theta$
$\mathbf{Var}(\tilde{\mathbf{R}})$	$(eL)^2\theta(\frac{1}{e} - \theta)$	$L^2\theta(\frac{1}{a} - \theta)$	$(eL)^2\theta(\frac{1}{ea} - \theta)$
Action affects:	Mainly on $E(\tilde{R})$	Only on $Var(\tilde{R})$	both
Reasons	Costs of efforts	Limited liability	both
First-Best	$L\theta = D'(e^{FB})$	$e = 1$	$L\theta = D'(e^{FB})$
	$a = 1$	$a^{FB} = \underline{a}$ $(D(a) > 0)$	a^{FB} = any value
			$(D(a) = 0)$
	$C^{FB} = 0$	$C^{FB} = 0$	$C^{FB} = 0$
Second-Best	$L\theta = D'(e^{SB}) + \Gamma(e^{SB})$	$e = 1$	$L\theta = D'(e^{SB})$
			$+\Gamma(e^{SB}, a^{SB})$
	$a = 1$	$a^{SB} = \underline{a}$	$a^{SB} = \underline{a}$ or $a^{SB} = \overline{a}$
	$C^{SB} > 0$	$C^{SB} = 0$	$C^{SB} > 0$
If $\Gamma > 0$:	$e^{SB} < e^{FB}$		$e^{SB} < e^{FB}$
			$a^{SB} = \underline{a}$
	$C^{SB} < r^{SB}$		$C^{SB} < r^{SB}$
If $\Gamma \leq 0$:	$e^{SB} \geq e^{FB}$		$e^{SB} \geq e^{FB}$
			$a^{SB} = \overline{a}$
	$C^{SB} \geq r^{SB} \geq 1$		$C^{SB} \geq r^{SB} \geq 1$

move in the same direction. This prediction is now testable using a similar setting as in Jimenez, Salas, and Saurina (2006) after controlling for the cost of collateral. Of course, to be more precise, we should use the ratio of collateral amount to loan size instead of a collateral dummy.

8.2 Testable Hypotheses

As shown in equation (5) in Section 6, the equilibrium collateral level is affected by effort (e), the marginal benefit of effort ($L\theta - D'(e^*)$) and the expected value ratio of collateral to the bank (ϕ^*). I use the borrower's average profitability ($E(\tilde{R}) - D(e)$) as a proxy of the product of effort and the marginal benefit of effort. ϕ^* is determined by both default risk ($1 - p^*$) and cost of collateral (β).

In Table 8.2, I summarize the variables in my model into two categories: unobservable and observable variables.

In the following, I develop two testable hypotheses based on my model of two-dimensional moral hazard. My model implies that collateral is determined by several characteristics of the borrower, including the borrower's quality type θ, his marginal productivity L, his marginal costs of effort $D'(e)$, and the social costs of collateral β. Taking all of these into account, I construct an empirical model on the use of collateral as follows:

Table 8.2: A Summary of Model Variables

Unobservable Variable		**Observable Variable**	
θ	Borrower's type	$1 - p(e, a, \theta)$	Default risk
e	Borrower's effort	C	Collateral amount over loan size
a	Borrower's action	r	Gross interest rate
	to control risk	$1 - \beta$	Cost of collateral
			(Firm-specific assets: more costly)
		$E(\tilde{R}) - D(e)$	Profitability
		$Var(\tilde{R})$	Volatility

$$
\begin{aligned}
C = \quad & F(b_0 + b_1 \cdot Default\_Risk + b_2 \cdot Profit\_R \\
& + b_3 \cdot I_{C \geq r} \times I_{firm-specific} + b'_3 \cdot I_{firm-specific} \\
& + b_4 \cdot I_{industry} + \text{Other Control Variables}), \qquad (8.1)
\end{aligned}
$$

where C is collateral amount over loan size, r is the gross interest rate, $Default\_Risk$ is the probability that the project fails (the proxy of which can be the borrower's credit rating), and $Profit\_R$ is the borrower's historical average profit over firm size. There are also three dummy variables in the formula. $I_{C \geq r}$ equals to 1 when $C \geq r$. $I_{firm-specific}$ equals to 1 when the collateral is the borrower's firm-specific assets (I use the classification of firm-specific assets as in Liberti and Mian (2010)). $I_{industry}$ is used to control for industry fixed effects. Now I present two hypotheses:

Hypothesis 1 *The use of collateral increases with the borrower's default risk ($b_1 > 0$), and decreases with the borrower's profitability ($b_2 < 0$).*

Hypothesis 2 *When C is no lower than r, a higher level of collateral is used when collateral is a firm-specific asset than when collateral is a non-firm-specific asset ($b_3 + b'_3 > 0$); When C is lower than r, a lower level of collateral is used when collateral is a firm-specific asset than when collateral is a non-firm-specific asset ($b'_3 < 0$).*

When the collateral is firm-specific, it is more likely to have a higher liquidation cost and thus implies a higher social cost of collateral. According to my model, a higher social cost of collateral leads to a lower level of collateral when $C < r$.

8.3 Limitations and Future Work

In the following, I discuss a few limitations of my model and propose future research directions.

First of all, I make some assumptions in my model in order to insure interior solutions of effort e. However, the range of effort choices might be bounded in practice. It can be shown that under certain boundaries of effort levels, zero collateral and the first-best effort level might sometimes be optimal in equilibrium. Boot, Thakor, and Udell (1991) is one extreme example. This implication offers a good explanation for the empirical fact that about 30% of the U.S. loans are unsecured. Boot, Thakor, and Udell (1991) also mention that the zero-collateral (for the good-type borrower) equilibrium can happen when the good-type borrower is more likely to default given that the borrowers choose their respective first-best actions.

Second, the case with a continuum of types should be non-trivial and deserve our further investigation given the contradicting conclusions in prior literature. Rothschild and Stiglitz (1976) discuss in their Footnote 7 about the non-existence of equilibrium due to the benefit of pooling.[5] However, Chan and Thakor (1987) argue that "the results with a continuum of types are mostly similar to those obtained here" (either for separating or for pooling). It is interesting to investigate the reason of this difference. Could it be attributed to the introduction of the moral hazard effects? In addition, the model in Chan and Thakor (1987) allows only two choices of actions for each borrower. In a simple model with only two types of borrowers, two choices of actions might be perfect to capture the two extreme cases. However, with a continuum of types, the limited choices of actions might rule out a large variety of contract set and lead to very different equilibria. Could that be the reason why Chan and Thakor (1987) find similar results with a continuum of types to those with only two types?

Third, with more than one banks in the credit market, the possibility of collusion when they interact repeatedly might lead to interesting results, especially when we consider the different kinds of equilibria: separating or pooling. For example, which type of contract is most conducive to collusion in equilibrium? Boot and Thakor (1994) have offered a dynamic model under shirking moral hazard. But a dynamic model with two-dimensional moral hazard and adverse selection is still an open question.

Fourth, most of the papers I cited here also discuss the possibility and impacts of credit rationing (e.g., Stiglitz and Weiss (1981), Wette (1983), Bester (1985), Stiglitz and Weiss (1986), Besanko and Thakor (1987a), Clemenz (1993), Besanko and Thakor (1993)), for it is an important dimension in a credit contract that generates a stream of literature. To avoid the credit-rationing problem and to focus on the incentive mechanism of collateral, I assume that banks have unlimited loan supplies and the borrower has unlimited access to collateral.

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[5]They claim that when there are relatively few low-types in the market, the principal might have incentive to deviate from a separating equilibrium to a pooling equilibrium for more profits. But similar to I have showed in Section 5.1.3, the pooling equilibrium is not sustainable. So in this case, no equilibrium exists. Not until recently does Guerrieri, Shimer, and Wright (2010) show that since firms can match with at most one worker, such a deviation cannot serve the entire population so that equilibrium always exists. An extension for the credit contract under asymmetric information is to compare the separating and pooling equilibrium, also taking into consideration the moral hazard effect and imperfect competition, and discuss whether both equilibria are sustainable and if so, which one is more efficient.

I argue that when risk-taking moral hazard is the only friction, the borrower provides zero collateral anyway even if these assumptions are relaxed. That is, the credit-rationing problem is just a second-order effect in this case. But when we also consider adverse selection, ex-ante, it is not clear to which direction credit rationing leads us.

Fifth, Besanko and Thakor (1987b) argue that variable loan size rules out credit rationing in some cases. Indeed, loan size can be another contract dimension to be included in my model. It could be a fruitful research direction to pursue implications in capital structure and other corporate governance issues. For example, the trade-off theorem of capital structure is closely related to the agency costs of debt that is due to risk-taking moral hazard.[6] Whereas the pecking-order theorem is closely related to the adverse selection problem. An extended model with optimal loan size might also inspire new thinkings in the debate of the capital structure puzzle.

However, one caveat is that because what I focus here is the optimal contract, I assume that a lot of firm characteristics are exogenously given. Nevertheless, the capital structure is determined simultaneously with other corporate decisions. For example, β is a constant parameter in my model. But from the borrower's perspective, it can be affected by monitoring, capital market environment, borrower's incentive to maintain the assets that are used as collateral and so on. That is, β might interact with other corporate governance variables (e.g., Leeth and Scott (1989), Gan (2007)) such as capital structure (e.g., James H. Scott (1977), Smith and Warner (1979)).[7] Therefore, conclusions need to be made with extra cautions.

Last but not least, in this thesis, I only consider the case when the borrower chooses a single investment project. In this case, risk-taking moral hazard is a simple version of the over-investment problem. However, the underinvestment problem is not considered because I assume a large enough L to insure contract and thus investment. If multiple investment projects are allowed, the underinvestment problem can be another type of moral hazard in addition to the over-investment problem (e.g., Stulz and Johnson (1985), Flannery, Houston, and Venkataraman (1993)).

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[6] In their well-known session of "agency costs of debt", Jensen and Meckling (1976) also describe risk-taking moral hazard in two examples: one with the "same expected total payoff" that is similar to my Section 5 and the other with negative NPV project that is similar to my Section 7.

[7] Roberts and Sufi (2009) find that firms with higher debt capacity and better financing sources are less likely to have an increased collateral requirement after a violation of debt covenant. Other papers that are related to capital structure and collateral include Morellec (2001) and Rauh and Sufi (1995).

APPENDIX

PROOFS

A.1 Proofs for Section 5

A.1.1 Robustness: Technology of Project Return

At the beginning of Section 5, I describe my technology of the project's return $\tilde{R}$ as:

$$\tilde{R} = \begin{cases} \dfrac{L}{a_i} & \text{with probability } a_i\theta_i \\ 0 & \text{with probability } 1 - a_i\theta_i \end{cases}$$

This MPS technology is distinct from prior literature mainly in the effects of type and action on the two major properties of the project's return: expected return and risk. Using FSD technique, Chan and Thakor (1987) assume that type and effort together can affect both properties of the project's return. Whereas with MSP technique, risk is affected by both type and action but expected return is solely determined by type. I choose this technology so that I am able to isolate the risk-taking effect from the shirking effect. However, it is interesting to investigate an even more fundamental case where risk is solely determined by action while expected return is solely determined by type. This intuitively simpler case in fact requires a more complicated model. In the following, by reverse-engineering a prevalent risk-return expression in asset pricing[1], I show that my results are still robust with the new assumption.

In fact, the two-outcome expression above is a special case of the following $\tilde{R}$:

$$\tilde{R}_i = L\theta_i + \tilde{\epsilon}_i, \text{ where } E(\tilde{\epsilon}_i) = 0 \text{ and } Var(\tilde{\epsilon}_i) = L\theta_i \cdot \frac{L}{a_i} \cdot (1 - a_i\theta_i).$$

With the new assumption, the variance then becomes $Var(\tilde{\epsilon}_i) = \dfrac{1}{a_i}$. To make is tractable in my model, we do reverse-engineering and turn it back to the zero-in-failure two-outcome case as follows:

[1] I would like to thank Professor Shmuel Baruch and Professor Hendrik Bessembinder for suggesting this alternative presentation.

$$\tilde{R} = \begin{cases} L\theta_i + \dfrac{1}{a_i L\theta_i} & \text{with probability } 1 - \dfrac{1}{a_i L^2\theta_i^2} \\ 0 & \text{with probability } \dfrac{1}{a_i L^2\theta_i^2} \end{cases}$$

With the new assumption, the expressions of V_i and Π_i remain the same as before with the new term $1 - \dfrac{1}{a_i L^2\theta_i^2}$ in replacement of $a_i\theta_i$. I have checked each step of the analysis in Section 5 and confirm that the new expression of probability of success does not affect the shapes or positions of the lines in any figure qualitatively. Because the key properties in my analysis that are related to this change are all about the slopes of indifference curves and those of zero-profit lines, and what matter to these slopes are their first-order partial derivatives with respect to type and to action. All else equal, either a higher a_i or a higher θ_i would make the slopes flatter no matter which expression we use of the probability of success. Therefore, all my results are robust with the new assumption that only action, but not type, affects project risk.

A.1.2 Proof of Lemma 3: Existence of Separating Equilibrium

In the following, I first derive the condition of λ that rules out alternative pooling equilibria, and then derive the condition λ that rules out alternative separating equilibria.

Condition of λ to Rule Out Alternative Pooling: In Figure 5.4 of Section 5.1.3, the (green) dash-dot lines are the market odds lines $\Pi(C_P, r_P; \bar{\theta}_\lambda) = 0$ for a pooling contract with different λs. A larger λ moves the market odds line upper and steeper. Because $\theta_B \le \bar{\theta}_\lambda \le \theta_G$ and the slope of the market odds line is $-\dfrac{\beta(1 - \underline{a}\bar{\theta}_\lambda)}{\underline{a}\bar{\theta}_\lambda}$, the market odds lines cross the good-type's zero-profit line at the point $r = \beta C < C$. That is, in the feasible region where $C < 1 \le r$, the market odds lines never cross the good-type borrower's zero-profit line at any point to the left of G. Therefore, with a large enough λ, the market odds line has an interception on r above G_1. That is, the market odds line lies above the good-type borrower's indifference curve, the line $G_1 G$[2] which G lies on. As a result, any pooling contract P' that is preferred by both types of borrowers makes a negative profit and would not upset the existent separating equilibrium.

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[2]For some large β, the slope of the market odds line might sometimes be even steeper than the good-type borrower's indifference curve that G lies on. However, this case only happens when the interception of the market odds line on r is below G_1, otherwise the market odds line would cross the good-type borrower's zero-profit line at a point that is on the left to G.

In order to find out the critical value for λ, I calculate the contract at G_1, and then use the zero-profit condition for pooling contract and zero collateral to get r_P. r_P that should be above G_1. First, because the indifference curve G_1G yields a utility of

$$V_G = l\theta_G - 1 - \frac{(1-\beta)(1-\underline{a}\theta_G)(\theta_G-\theta_B)}{\theta_G - \theta_B[\beta + (1-\beta)\underline{a}\theta_G]} - \frac{1}{2}c\underline{a}^2$$

as shown in equation (5), at the interception on r with $C_{G_1} = 0$, we have:

$$r_{G_1} = \frac{1}{\underline{a}\theta_G} \cdot \frac{(1-\beta)(1-\underline{a}\theta_G)\theta_G + (\theta_G - \theta_B)}{\theta_G - \theta_B[\beta + (1-\beta)\underline{a}\theta_G]}.$$

Second, with $\Pi(0, r_P; \bar{\theta}_\lambda) = 0$, we have $r_P = \dfrac{1}{\underline{a}\bar{\theta}_\lambda}$. Finally, because the market odds line with λ lies above G_1G, or $r_P > r_{G_1}$, we have:

$$\lambda > \frac{(1-\beta)(1-\underline{a}\theta_G)\theta_G}{(1-\beta)(1-\underline{a}\theta_G)\theta_G + \theta_G - \theta_B}.$$

Condition of λ to Rule Out Alternative Separating: In Section 5.1.3, I have explained that subsidy story. If on average, a bank can make a non-negative profit from it, the alternative set of contracts B' and G' as shown in Figure 5.4 might upset the existing separating contracts. In this proof, I demonstrate in details how a large enough λ can rule out this possibility.

First of all, Section 5.1.1 has shown that zero collateral is always optimal for the bad-type borrower and thus B' must also be on the r axis. Second, for B' and G' to upset the existing separating equilibrium, H' must fulfill the following conditions: (1) G' lies above the bad-type borrower's indifference curve (the one that passes B') so that the bad-type borrower does not mimic the good-type; (2) G' lies below the good-type borrower's indifference curve G_1G so that both types of borrowers are better off; (3) The average profit from B' and G' is non-negative.

Because B' lies below the bad-type borrower's zero-profit line, it makes a negative profit for a bank. In order to subsidize the loss with the good-type contract G', G' must lie above the good-type borrower's zero-profit line G_0G. Conditions (1) and (2) imply that any potential contract G' is restricted in the small triangle that is formed by the bad-type borrower's indifference curve (the one that passes B'), the good-type's indifference curve G_1G and the zero-profit line G_0G. Mathematically, we have:

$$\begin{cases} V(C_G', r_G'; \theta_B, \underline{a}) \leq V(0, r_B'; \theta_B, \underline{a}) \text{ or } \dfrac{r_B' - r_G'}{C_G'} \leq \dfrac{1 - \underline{a}\theta_B}{\underline{a}\theta_B} \\ V(C_G', r_G'; \theta_G, \underline{a}) \geq V(C_G, r_G; \theta_G, \underline{a}) \text{ or } \dfrac{r_G' - r_G}{C_G - C_G'} \leq \dfrac{1 - \underline{a}\theta_G}{\underline{a}\theta_G} \\ \lambda\Pi(0, r_B'; \theta_B, \underline{a}) + (1-\lambda)\Pi(C_G', r_G'; \theta_G, \underline{a}) \geq 0 \\ \qquad \text{or } \lambda\underline{a}\theta_B r_B' + (1-\lambda)[\underline{a}\theta_G r_G' + (1-\underline{a}\theta_G)\beta C_G'] \geq 1 \end{cases}$$

where

$$C_G = \frac{\theta_G - \theta_B}{\theta_G - \theta_B[\beta + (1-\beta)\underline{a}\theta_G]} \text{ and } r_G = \frac{\beta\theta_G + (1-\beta)\frac{1}{\underline{a}} - \theta_B}{\theta_G - \theta_B[\beta + (1-\beta)\underline{a}\theta_G]}.$$

To find the critical value of λ_0 that on average makes a zero profit for a bank, I solve the set of simultaneous equations above with each constraint binding (by treating r_B' as known, we have three equations and three unknown variables: r_G', C_G' and λ). The result is:

$$\lambda_0 = \frac{(1-\beta)(1-\underline{a}\theta_G)\theta_G}{(1-\beta)(1-\underline{a}\theta_G)\theta_G + \theta_G - \theta_B}.$$

This result suggests that r_B' is cancelled out in the calculation. Thus, the expression of λ_0 is exactly the same as the critical value of λ in condition (1) that rules out alternative pooling. With any $\lambda > \lambda_0$, the losses from the bad-type contracts are larger with more bad-type contracts so that banks make negative profits on average. Therefore, as long as $\lambda > \lambda_0$, there is no alternative separating contract set that would upset the equilibrium contracts B and G.

A.2 Proofs for Section 7

A.2.1 Proof of Lemma 4:

In this maximization problem of e, what matters to the properties of the solutions is the concavity of $V(e,\theta)$. It is determined by the second-order derivative. As long as $\dfrac{d^2V(e,\theta)}{de^2} \leq 0$ for any e such that $\dfrac{dV(e,\theta)}{de} \geq 0$ and $V(e,\theta)$ is a continuous function of e, the optimal solution of e must be (1) at the corners, or (2) among the stationary points (where $\dfrac{dV(e,\theta)}{de} = 0$).

For (1), because $\dfrac{dV(e,\theta)}{de}|_{e=0} > 0$, we can rule out the corner solution at $e = 0$. And with $D'(\overline{e}) = +\infty$ in Assumption ($A2b$), $\dfrac{dV(e,\theta)}{de}|_{e=\overline{e}} < 0$, so we can also rule out $e = \overline{e}$. So we can rule out case (1).

As also stated in Assumption $(A3b)$, $D''(e) \geq 0$ and $D'''(e) \geq 0$. So with Assumption $(A3b)$, the second part of $\frac{d^2V(e,\theta)}{de^2}$ (see equation (SOC_c)) is always non-positive. Therefore, when $\frac{dV(e,\theta)}{de} \geq 0$, $\frac{d^2V(e,\theta)}{de^2} \leq 0$.

As a consequence, the equilibrium e can be found by comparing the corresponding objective functions for the stationary points that are obtained by using the "first-order approach" (one if $L\theta < D'(e)$ and one or more than one if $D'''(e) > 0$ and $L\theta \geq D'(e)$).

A.2.2 Proof of Lemma 5:

Consider all the possible shapes of the objective function over e, one can easily show that as long as $\frac{d^2V(e,\theta)}{de^2} \leq 0$ for any e, the objective function is a hump shape with only one stationary point.

(SOC_c) can be rewritten as:

$$\frac{d^2V(e,\theta)}{de^2} = -\frac{2(1-\beta)\cdot p_e[\beta(L\theta - D'(e)) + (1-\beta)p_e]}{\phi^3(e)}$$
$$-\frac{(2\beta - \phi^2(e))\cdot D''(e) + \phi(e)(1-\phi(e))\cdot eD'''(e)}{\phi^2(e)}$$

To insure a non-positive second-order derivative for any e, we need to have:

$$D'(e) \leq L\theta + \frac{2(1-\beta)p_e}{\beta} + \frac{\phi(e)(2\beta - \phi^2(e))D''(e) + \phi^2(e)(1-\phi(e))eD'''(e)}{2\beta(1-\beta)p_e}.$$

A.2.3 Proof of Lemma 11:

This proof is similar to the proof of Lemma 4. But now with risk-taking moral hazard, $V(e,\theta)$ is not a continuous function of e. It has a discontinuous point at $e = e^{FB}$ (where $L\theta = D'(e)$). Correspondingly, there are three potential solutions of e: (1) at the corners, or (2) at the discontinuity points, or (3) among the stationary points (where $\frac{dV(e,\theta)}{de} = 0$).

Same as in the proof of Lemma 4, (1) can be easily ruled out.

Again, with Assumption $(A3b)$, the second part of $\frac{d^2V(e,\theta)}{de^2}$ (see equation (SOC)) is always non-positive. Therefore, when $\frac{dV(e,\theta)}{de} \geq 0$, $\frac{d^2V(e,\theta)}{de^2} \leq 0$. And the optimal solution of e must be either (2) at the discontinuity points or (3) among the stationary points.

For (2), according to (IC'_a), there is only one discontinuity point at $e = e^{FB}$ where $L\theta = D'(e)$.

As a consequence, the equilibrium e can be found by comparing the corresponding objective functions under second-best for $e = e^{FB}$ and for the stationary points that are obtained by using the "first-order approach" (one such that $L\theta < D'(e)$ and one or more than one if $D'''(e) > 0$ such that $L\theta \geq D'(e)$).

A.2.4 Proof of Lemma 12:

This proof is similar to the proof of Lemma 5. But now with risk-taking moral hazard, as long as $\dfrac{d^2V(e,\theta)}{de^2} \leq 0$ for any e, there are only two peak values of e.

1. Because $\dfrac{dV(e,\theta)}{de}|_{e=0} > 0$, the part in the region of $L\theta < D'(e)$ is a hump shape.
2. As stated in Assumption $(A3b)$, $D'(\overline{e}) = +\infty$ so that $\dfrac{dV(e,\theta)}{de}|_{e=\overline{e}} < 0$.
   If $\dfrac{dV(e,\theta)}{de}|_{e=e^{FB},a=\overline{a}} \geq 0$, the part in the region of $L\theta \geq D'(e)$ is another hump shape; otherwise, it is the right part of another hump shape with a peak value at point $e = e^{FB}$.

(SOC) can be rewritten as:

$$\frac{d^2V(e,\theta)}{de^2} = -\frac{2(1-\beta)\cdot p_e(a^*,\theta)[\beta(L\theta - D'(e)) + (1-\beta)p_e(a^*,\theta)]}{\phi^3(e)}$$
$$-\frac{(2\beta - \phi^2(e))\cdot D''(e) + \phi(e)(1-\phi(e))\cdot eD'''(e)}{\phi^2(e)}$$

To insure a non-positive second-order derivative for any e, we need to have:

$$D'(e) \leq L\theta + \frac{2(1-\beta)p_e(a^*,\theta)}{\beta} + \frac{\phi(e)(2\beta - \phi^2(e))D''(e) + \phi^2(e)(1-\phi(e))eD'''(e)}{2\beta(1-\beta)p_e(a^*,\theta)}.$$

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